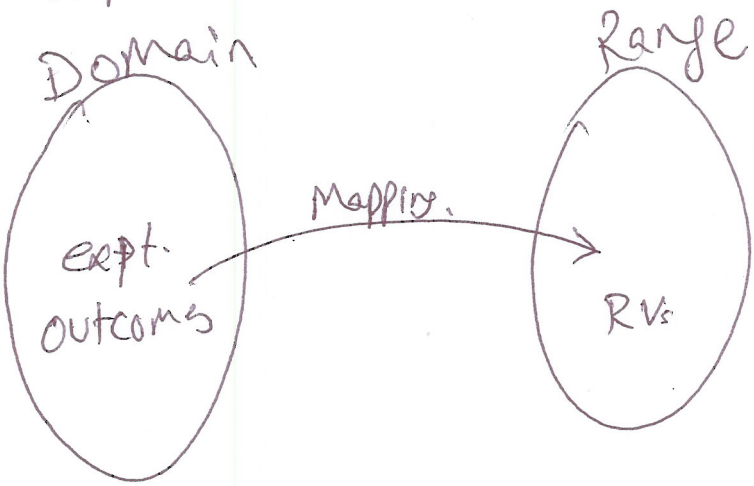
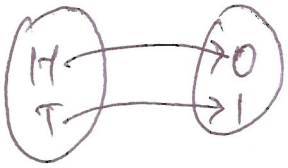
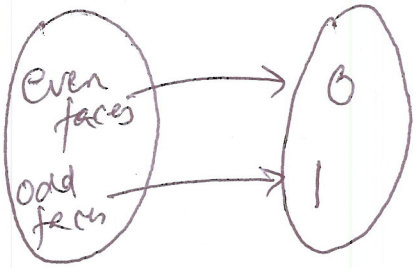
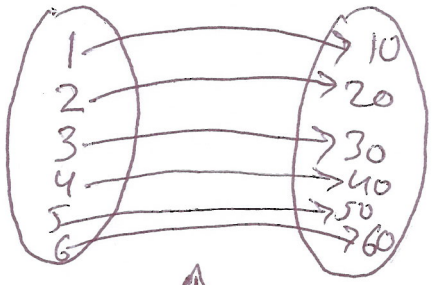
Random Variables (RV)

RV is a number assigned to every outcome of an experiment.









~~$x(i) = 0$~~

$x(i) = 10\,i$

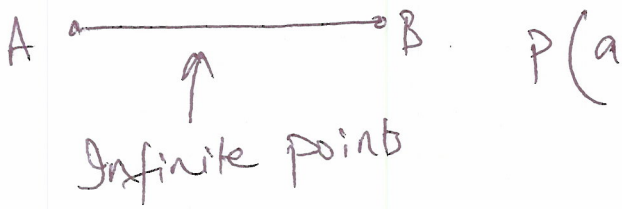
$P(x \leq 35) = \frac{3}{6} = 1/2$

$P(x \leq 5) = 0$

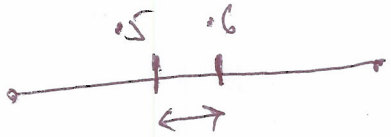
$P(x = 40) = \frac{1}{6}$

The Real Line

If S consists of infinite elements, its prob. cannot be determined by the prob. of its elementary elements.



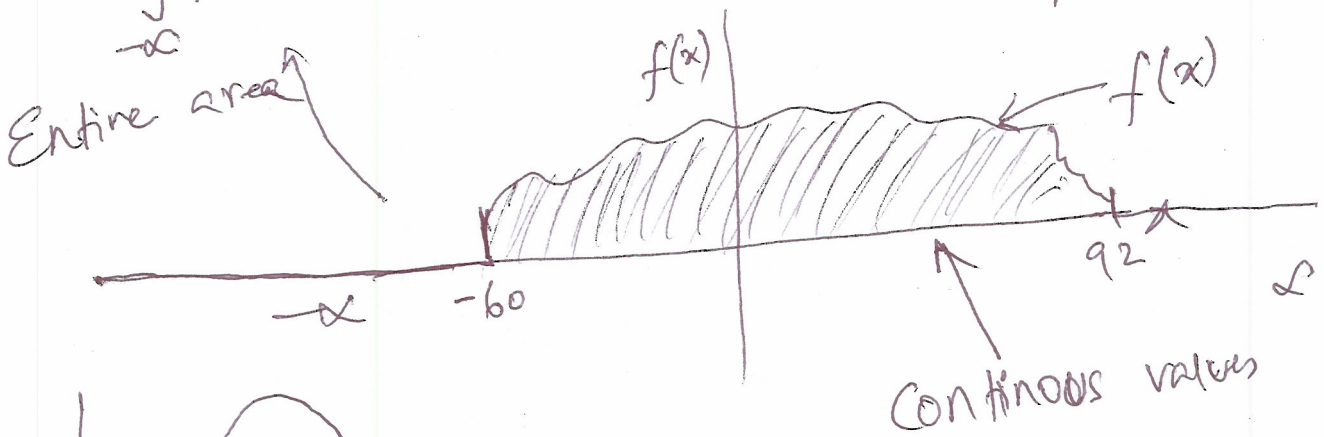
$P(\text{any point on line } AB) = \frac{1}{\infty}$ Not defined

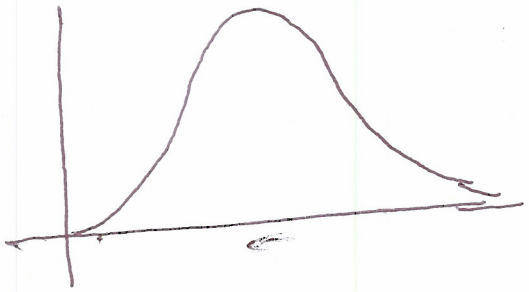


Probability Density function (Pdf)

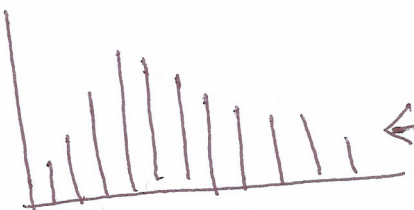
Suppose $f(x)$ is a function such that

$$\int_{-\infty}^{\infty} f(x)\,dx = 1$$ then $f(x)$ is a Pdf of RV x.



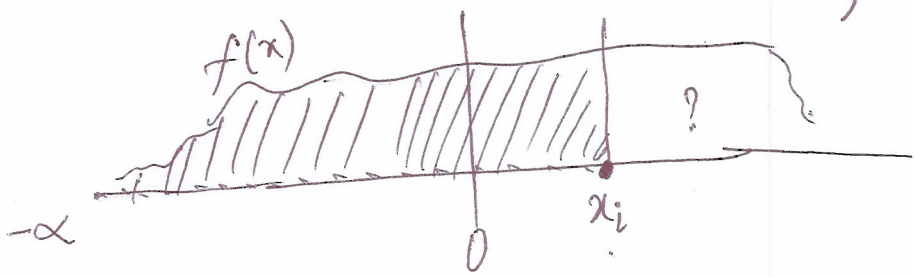


$\sum_i P_i = 1$



Discrete RVs.

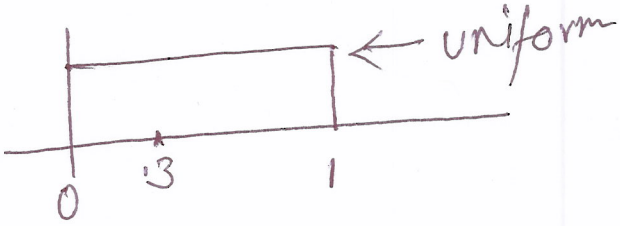
PMF Probability mass function

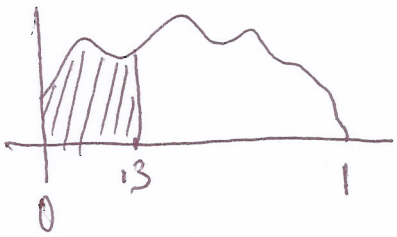


$$P(x \le x_i)$$

$$= \int_{-\infty}^{x_i} f(x)\,dx$$

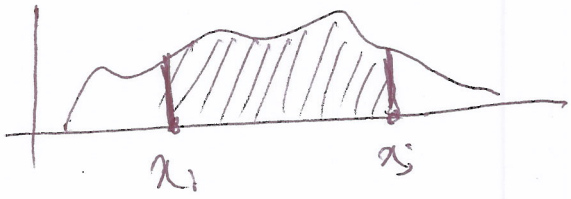
$$= 1 - \int_{x_i}^{\infty} f(x)\,dx$$





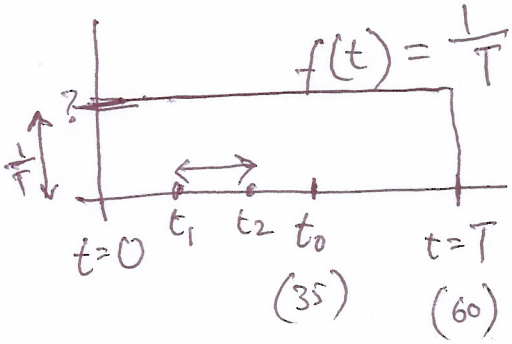
$$P(x_i < x \le x_j)$$

$$= \int_{x_i}^{x_j} f(x)\,dx$$



Prob A telephone call is made randomly (uniform) in $(0, T)$.

$$\text{Prob}\left(\begin{array}{l}\text{call arrives in the}\\ \text{interval } 0 \le t \le t_0\end{array}\right) = \frac{t_0}{T}$$



$$\text{Prob}\left(\text{call in } t_1 \le t \le t_2\right) = \frac{t_2 - t_1}{T}$$

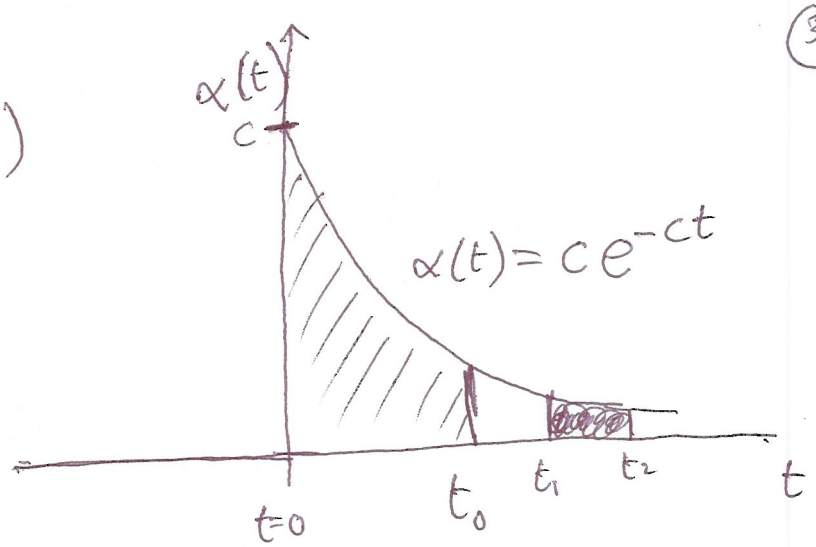
$$\text{Prob}\left(\text{call in } 0 \le t \le T\right) = 1$$

Example

$$\alpha(t) = c e^{-ct}\, u(t)$$

$u(t) = 1$ for $t \geq 0$

$u(t) = 0$ for $t < 0$



$$P(0 \leq t \leq t_0) = \int_0^{t_0} \alpha(t)\,dt = \int_0^{t_0} c e^{-ct}\,dt = c \cdot \frac{e^{-ct}}{-c}\Big|_0^{t_0}$$

$$= -e^{-ct}\Big|_0^{t_0}$$

$$= -e^{ct_0} - (-e^0)$$

$$= \underline{1 - e^{ct_0}}$$

Cumulative Distribution Function (cdf)

$$F(x) = P(\mathcal{X} \leq x)$$

eg die expt.

$\mathcal{X}(f_i) = 10i \Rightarrow f_i \{10, 20, 30, 40, 50, 60\}$

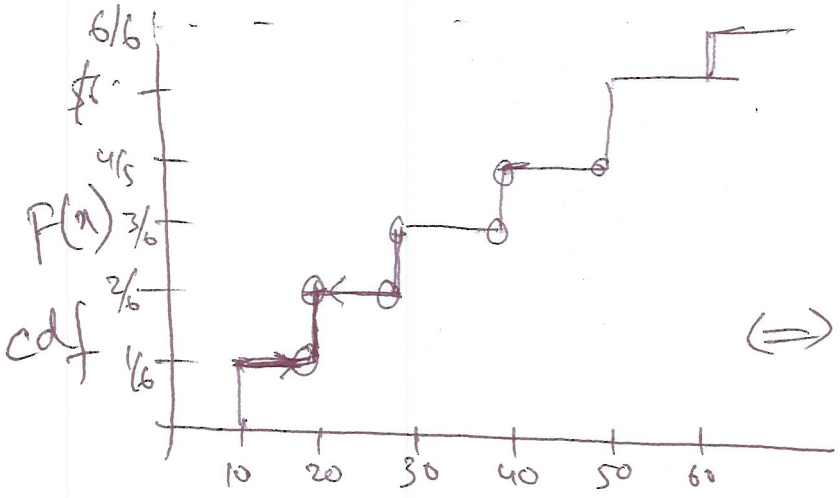
$F(100) = P\{\mathcal{X} \leq 100\} = 1$

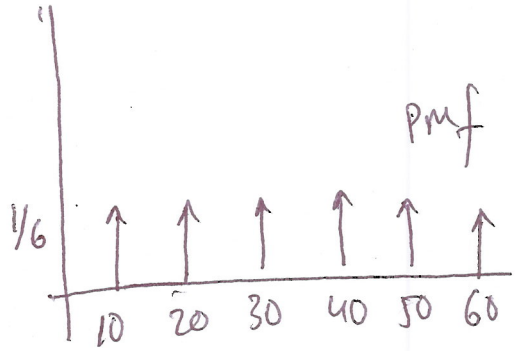
$F\{60\} = P\{\mathcal{X} \leq 60\} = 6/6 = 1$

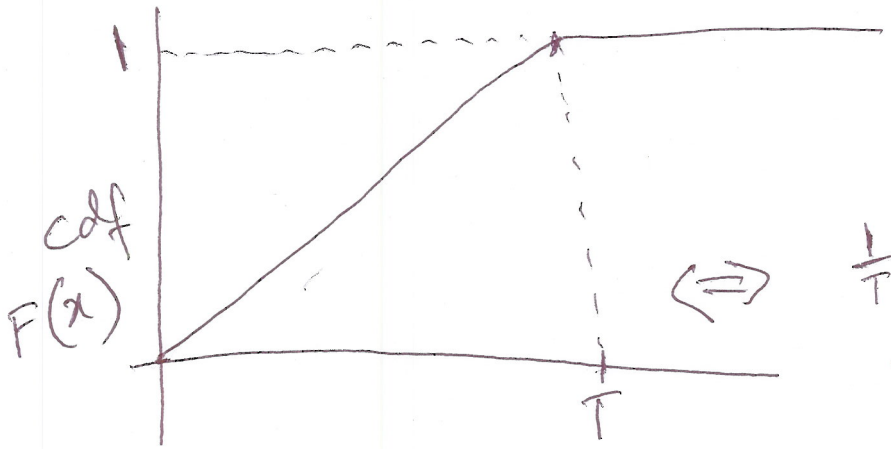
$F\{59\} = P\{\mathcal{X} \leq 59\} = 5/6$

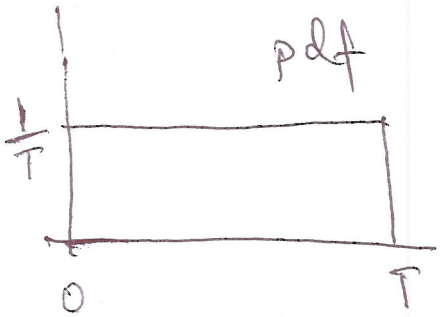
$F\{30.1\} = P\{\mathcal{X} \leq 30.1\} = 3/6$

$F\{30\} = P\{\mathcal{X} \leq 30\} = 3/6$









pdf cdf

$$f(x) = \frac{d}{dx} F(x)$$

$$F(x_2) = \int_0^{x_2} f(x)\,dx = P\{x \le x_2\} \quad -(1)$$

$$F(x_1) = \int_0^{x_1} f(x)\,dx = P\{x \le x_1\} \quad -(2)$$

$F(6'0'')$
$F(7'2'')$

$x_2 > x_1$

$$F(x_2) - F(x_1) = \int_{x_1}^{x_2} f(x)\,dx \Rightarrow \text{Eqn (2)} - \text{Eqn (1)}$$