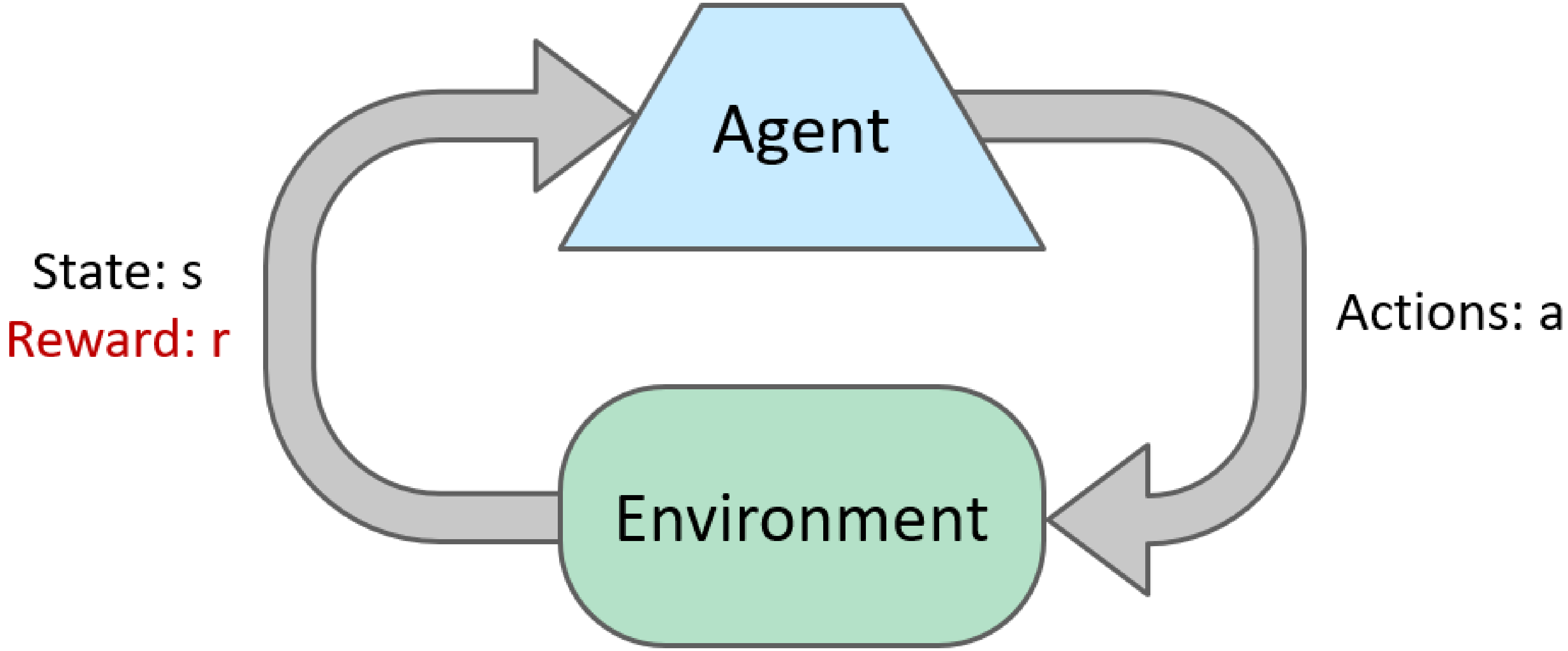
Policy gradient reinforcement learning

Some material on these slides are derived from the slides created by David Silver for University College London, available at https://davidstarsilver.wordpress.com/ .


Agent
State: s
Reward: r
Actions: a
Environment

RL model

- We assume a world governed by an MDP
  - States $s \in S$
  - Actions A
  - System dynamics $T(s, a, s')$
  - Reward function $R(s, a, s')$
- We are looking for the policy $\pi(s)$
- But we don't know T and/or R as functions
- But if we take an action a, in state s, we can observe that we landed in s' and received reward r.

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- What we have done until now is primarily value-based RL
  - We estimated values: $V(s)$, $Q(s,a)$
- Let us introduce a new value, the **advantage**
  - It shows how much a certain action is worse than the optimal action

$$A(s,a) = Q(s,a) - V(s)$$

- The whole point of Q-learning was to calculate the Q-value without having to estimate the whole MDP (the transition function T, the reward function R)
  - We estimated the Q by getting sample rewards as we were acting in the world
- Then, we used the Q value to infer the policy π

Policy gradient RL

- Can we estimate directly the policy π without going through the Q value first?
- The idea is that we have a parameterized policy $\pi_\theta(s, a)$
    - Find the best θ
    - Based on the rewards sampled from the environment.
- How do we measure the quality of the policy π_θ?
- We will define a function $J(\theta)$ that describes the quality of the policy

Describing the quality of a policy $\pi(\theta)$

- In discrete environments we can use the **start value**

$$J_1(\theta) = V^{\pi_\theta}(s_1) = \mathbb{E}_{\pi_\theta}[v_1]$$

- In continuing environments we can use the **average value**

$$J_{avV}(\theta) = \sum_s d^{\pi_\theta}(s) V_{\pi_\theta}(s)$$

- Or the **average reward per time-step**

$$J_{avV}(\theta) = \sum_s d^{\pi_\theta}(s) \sum_a \pi_\theta(s, a) R_s^a$$

- Where $d^{\pi_\theta}(s)$ is the **stationary distribution of the Markov chain** for π_θ
  - Basically: how much time are you going to spend in state s if you follow in π_θ?

Policy optimization

- Policy-based reinforcement learning is an optimization problem
  - Find θ that optimizes $J(\theta)$
- There are many approaches one can use, and not all of them use the gradient
  - Hill climbing
  - Simplex / amoeba / Nelder Mead
- Greater efficiency often possible using gradient descent
  - We will focus on that.

Policy gradient

- Let $J(\theta)$ be any policy objective function.
- **Policy gradient algorithms** search for a local maximum of $J(\theta)$ by ascending the gradient of the policy, w.r.t. parameters θ

$$\Delta\theta = \alpha \nabla_\theta J(\theta)$$

where $\Delta_\theta J(\theta)$ is the **policy gradient** and α is a step-size parameter

$$\nabla_\theta J(\theta) = \begin{bmatrix} \frac{\partial J(\theta)}{\partial \theta_1} \\ \frac{\partial J(\theta)}{\partial \theta_2} \\ \vdots \\ \frac{\partial J(\theta)}{\partial \theta_n} \end{bmatrix}$$

Computing gradients by finite differences

- To evaluate the policy gradient of $\pi_\theta(s, a)$
- For each dimension $k \in [1, n]$
  - Estimate the k-th partial derivative of the objective function wrt to θ
  - By perturbing θ with a small amount ϵ in the k-th dimension

$$\frac{\partial J(\theta)}{\partial \theta_k} \approx \frac{J(\theta + \epsilon u_k) - J(\theta)}{\epsilon}$$

- where u_k is the unit vector in the k-th component, 0 elsewhere
- We need to evaluate J n times for one step.
- Simple, noisy, inefficient - but sometimes effective.
- Works for arbitrary policies, even if the policy is not differentiable.

Computing the gradient analytically

- Directly calculating the objective function J is a very big pain in the neck
  - Even for the definition with the value of the start state... that is basically solving the MDP! Which we don't have!
  - And with the density function: that assumes a level of knowledge about the environment that we don't have
- So we want some way to evaluate the policy gradient without actually calculating J
- Let us assume that the policy π_θ is differentiable whenever it is non-zero

Policy gradient theorem

- Applies to start state objective, average reward and average value objective.

Theorem

For any differentiable policy $\pi_\theta(s,a)$, for any of the policy objective functions $J = J_1$, J_{avR} or $\frac{1}{1-\gamma} J_{avV}$, the policy gradient is:

$$\nabla_\theta J(\theta) = \mathbb{E}\left[\nabla_\theta \; log(\pi_\theta(s,a)) \; Q^{\pi_\theta}(s,a)\right]$$

Why is the policy gradient a big deal

- We don't have to take the gradient of the J function, we don't even have to calculate it.
- We need to calculate the gradient of the log policy - and the policy function is something that we know, because we created it.
  - Eg. it is the neural network implementing the policy.
  - And we can just use the automatic differentiator
- It works both for discrete and continuous states and actions.
- We still need a way to estimate $Q^{\pi_\theta}(s, a)$
  - Note that it is the Q of the policy, not the perfect Q^*
  - Still, how do we estimate it?

REINFORCE

- The grandfather of all policy gradient approaches (Ronald Williams, 1992)
- Idea: estimate $Q^{\pi_\theta}(s_t, a_t)$ with the **return** v_t
    - That is: consider a run that terminates in state s_T. Add up all the future rewards between current timestep t to the end: this is v_t.
    - This is a single, unbiased sample of the Q-value at that point.
- Also called
    - "vanilla policy gradient" - no other tricks in this algorithm
    - "Monte Carlo policy gradient" - because it waits to the end of the run to calculate the total reward, and uses that as a learning signal.

REINFORCE algorithm

```
function REINFORCE
   initialize theta arbitrarily
   for each episode (s1,a1, r2, ...sT−1, aT−1, rT) do
      for t=1 to T−1 do
         theta = theta + alpha ∗ nabla log pi_theta (st,at) ∗ vt
      end for
    end for
end function
```

Why is REINFORCE not the perfect solution

- We need to wait to the end of the run
  - So we need to wait to see how the run turns out before we can update the policy
  - In Q-learning, we were learning at every step
- **High variance**
  - The return of a particular run can be very different from the Q value
  - Maybe you were just unlucky... but now you changed the policy away from the correct one...
- **On-policy algorithm**
  - We need to run the current policy to gather data.
  - Compare this to Q-learning which is off-policy: you can use samples from any other agent running any policy, and learning will still work.

Reducing variance using a critic

- We can use a **critic** to estimate the action-value function of the current policy:

$$Q_w(s, a) \approx Q^{\pi_\theta}(s, a)$$

- Actor critic algorithms maintain *two* sets of parameters:
  - **Critic** updates action-value function parameters w
  - **Actor** updates policy parameters θ, in the direction suggested by the critic
- Actor-critic algorithms follow an *approximate* policy gradient:

$$\nabla_\theta J(\theta) \approx \mathbb{E}_{\pi_\theta}\left[\nabla_\theta\, log\left(\pi_\theta(s, a)\right) Q_w(s, a)\right]$$
$$\Delta\theta = \alpha \nabla_\theta\, log\left(\pi_\theta(s, a)\right) Q_w(s, a)$$

Reducing variance using a baseline

- We subtract a baseline function $B(s)$ from the policy gradient
- This can reduce variance without changing expectation:

$$\mathbb{E}_{\pi_\theta}[\nabla_t log\ \pi_\theta(s,a)B(s)] = \sum_{s\in S} d^{\pi_\theta}(s) \sum_{a\in A} \nabla_\theta \pi_\theta(s,a)B(s)$$

We can take out the B(s) from the ∇_θ because it does not depend on θ

$$= \sum_{s\in S} d^{\pi_\theta}(s)\ B(s)\nabla_\theta \sum_{a\in A} \pi_\theta(s,a)$$

the sum for all a is 1, because one of the actions will be taken

$$= \sum_{s\in S} d^{\pi_\theta}(s)\ B(s)\nabla_\theta 1 = 0$$

Reducing variance using a baseline

- This means that we can subtract any function from the function we are taking the gradient on, as long as it depends only on the state, not on the action.
- A good baseline is the state value function $B(s) = V^{\pi_\theta}(s)$
- So we can rewrite the policy gradient using the **advantage function**

$$A^{\pi_\theta}(s,a) = Q^{\pi_\theta}(s,a) - V^{\pi_\theta}(s)$$
$$\nabla_\theta J(\theta) = \mathbb{E}_{\pi_\theta}\left[\nabla_\theta \, log \, \pi_\theta(s,a) A^{\pi_\theta}(s,a)\right]$$

- This creates the family of algorithms called **advantage actor critic** (A2C/A3C). Can significantly reduce the variance of policy gradient: faster and more stable convergence.