Expectimax search

Uncertain outcomes

- Single agent search tree assumed that you are the only agent taking actions and their result is predicted by the transition function $T(s, a) \to s'$
- Minmax assumed that there is also an opponent taking actions
  - But, in some sense, the zero sum opponent is also predictable
- What if we don't know the result of the action?
  - Inherent randomness in the environment: rolling a dice
  - Unpredictable opponents or bystanders with random behavior
  - Actions can fail or succeed partially (slipping wheels etc)

Probabilistic transition function

- A way to think about this is that the transition function is now probabilistic: $T(s, a, s') \in [0, 1]$
- We will talk more about this when we discuss Markov Decision Processes and reinforcement learning

Expectimax search

- We are still trying to compute the V value
- **max** nodes: return the max of successors
- **min** nodes: return the min of the successors
- **expectation** nodes: return the probability weighted average (*expectation*) of children
  EXAMPLE HERE

Reminder: expectation of a random variable

$$\mathbb{E}(f(x)) = \sum_i p(x_i) f(x_i)$$

or, in a continuous case:

$$\mathbb{E}(f(x)) = \int p(x) f(x) dx$$

Expectation of time to get to the airport:

- Drive time: 20 clear weather, 40 min in rain, 60 min in snow
- Likelihood of clear weather 80%, rain 15%, snow 5%
- Expectation: 20 * 0.8 + 40 * 0.15 + 60 * 0.05 = 25

Backgammon (or other dice-based zero-sum games)

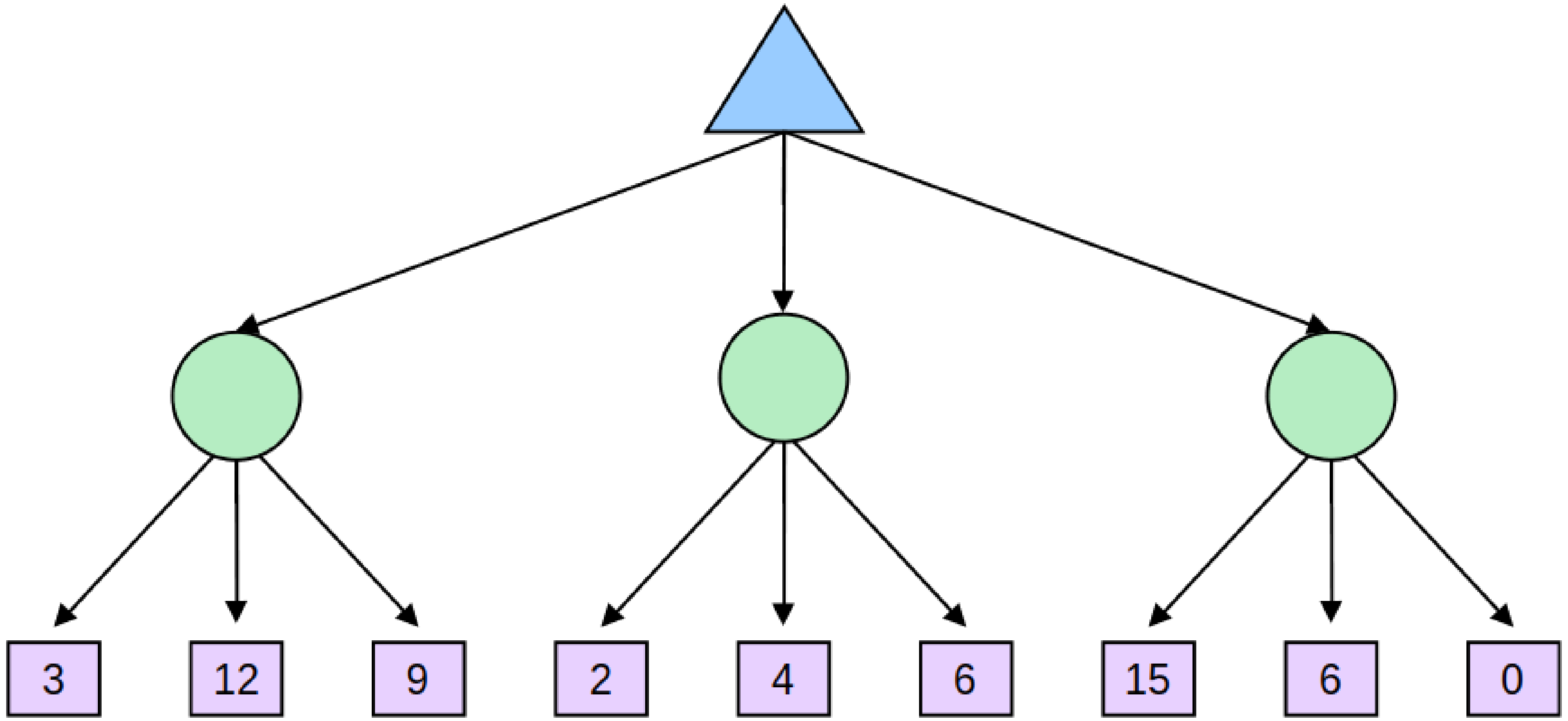
- Max node (move by ego, after knowing dice)
- Expect node (dice by opponent)
- Min node (move by opponent, after knowing dice)
- Expect node (dice by ego)
- Max node (move by ego, after knowing dice)... and so on

Expectimax (and other variants) pseudocode

```
def value(state):
  if state is TERMINAL: return value
  if state is MAX: return maxvalue(state)
  if state is MIN: return minvalue(state)
  if state is EXPECT: return expvalue(state)

def maxvalue(s)
  v = -∞
  for s' in succ(s)
    v = min (v, maxvalue(s'))
  return v

def expvalue(s)
  v = 0
  for s' in succ(s)
    v = v + probability(s') * value(s')
  return v
```


3
12
9
2
4
6
15
6
0

Expectimax pruning

- Can we prune expectimax?
- Problem: expectation can go both up and down with new nodes!
- Might involve a tricky computation, done probabilistically.

Depth limited expectimax

- Expectimax nodes can really blow up the computation time, because you need to evaluate everything below
- It is useless to make long plans when they depend on repeated dice throws to come out just so:
    - I will throw an 8 and move like this, then my opponent will throw a 4 and move like that, then I will throw an 11...
- Game programs for games with significant random component:
    - Think ahead only 1..4 plies
    - Use a **very good** evaluation function

Where do we get the probabilities from?

- In expectimax search, we need to know the probabilities of outcomes
  - Sometimes it is some uniform or near randomness (eg. dice)
  - Sometimes it is a small uncertainty on a positive or negative action.
- Where do we get the probabilities? - The model
  - Sometimes it is simple - eg. dice roll
  - Sometimes it is very complex
- We will revisit this later

Informed probabilities

- Expectimax can also handly situations where you try to model an imperfect opponent:
- Let us say that the opponent is doing the perfect minmax move 90% of the time, but moves randomly 10% of the time
- This is an expectation node. But you don't know the probability of the move ahead of time, you need to calculate it!
- You need to run a **simulation of your opponent**, with the opponent **simulating you**
  - This is very expensive for expectimax
  - It is much cheaper for minmax, because the two simulations are folded into the same tree.

Mixed layers

- Different layers (max/min/expectations) can be mixed randomly.
- Often, we consider the environment an additional "random" player.
- Each node computes the appropriate combination of its children.

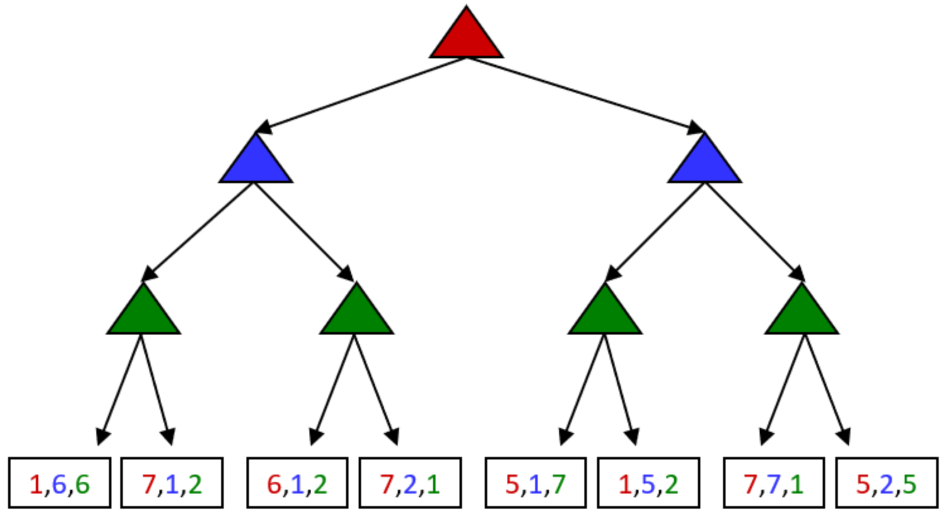
Example: Backgammon

- Dice rolls increase *b*: 21 possible rolls with 2 dice
  - Backgammon ≈ 20 legal moves
  - Depth 2 = 20 x (21 x 20)^3 = 1.2 x 10^9
- As depth increases, probability of reaching a given search node shrinks
  - So usefulness of search is diminished
  - So limiting depth is less damaging
  - But pruning is trickier…
- Historic AI: TDGammon uses depth-2 search + very good evaluation function + reinforcement learning: world-champion level play
- 1st AI world champion in any game!

Image: Wikipedia

Multi-agent games

- What if the game is not zero sum, or has multiple players?
- Node values are not tuples of utility
- Each player maximizes its own utility
- Emergent cooperation and competitition

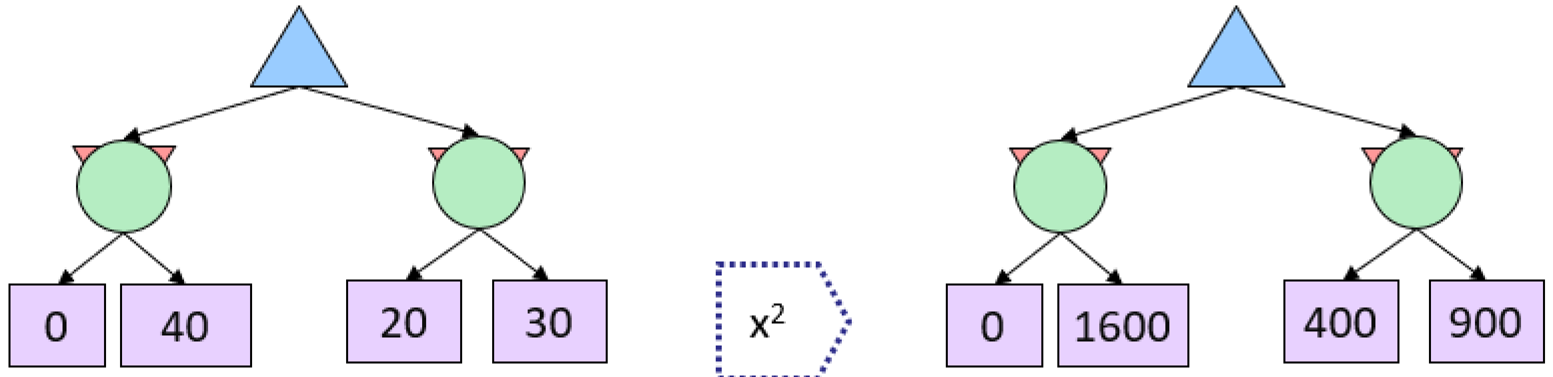


Utilities and rationality

- We kept talking about **utility** but did not yet give a good definition what it is
  - Let us say that the values associated with the terminal outcomes are the utility
  - Agents pursue higher utility, so these values determine the behavior of the agent
- We say that a **rational agent** is one that chooses actions that maximize its **expected utility**, given its knowledge

"Expected"?

- For minmax reasoning, utility is simple. The greater the utility outcome, the better.
    - So only **ordering** matters.
    - "insensitivity to monotonic transformations"
- But for expectimax, we take expectations, so suddenly the absolute values matter as well.
    - We should define utilities such that we can add and average them!


0
40
20
30
x²
0
1600
400
900

Utilities and preferences

- We define utilities as functions on states $U(s) \in \mathbb{R}$
- We will say that the utilities describe the **preferences of the agents**
  - It is a way to summarize the goals of the agent
- Two strategies to build an intelligent agent
  - **Behavior specification:** Describe its behavior for each state - i.e. write the $\pi(s) \rightarrow a$ function directly.
  - **Utility specification:** Provide a utility function $U(s)$. A rational agent will choose its own actions in the pursuit of the goal of maximizing expected utility.

Problems

- Problems with behavior specification:
  - **Framing problem:** need to handle a very large number of cases
    - Buy milk, unless you already have it, it is 2am, it is a hurricane, it is a zombie attack, ...
  - The relationship between the behavior and good outcomes difficult to prove.
- Problems with utility specification:
  - Where does the utility coming from?
  - Can every rational behavior be expressed as utilities?
  - **Theorem:** any rational preferences over states can be summarized as a utility function.

Preferences

- Prizes: A, B
- Lotteries: situations with uncertain prizes: $L = [p, A; (1-p)B]$
- The agent prefers A denoted by $A \succ B$
  - A good way to think about it is that the agent would pay at least $0.01 to get A instead of B
- The agent is **indiferent** denoted by $A \approx B$

Rational preferences

- What kind of preferences can be considered rational?
- Let us imagine an agent with $A \succ B$, $B \succ C$, $C \succ A$.
- Such an agent can be induced to give away all his money!
  - This happens because the preferences are not transitive
  - "Dutch book" auctions in horse races.

Orderability

$(A \succ B) \vee (B \succ A) \vee (A \sim B)$

Transitivity

$(A \succ B) \wedge (B \succ C) \Rightarrow (A \succ C)$

Continuity

$A \succ B \succ C \Rightarrow \exists p \; [p, A; \; 1-p, C] \sim B$

Substitutability

$A \sim B \Rightarrow [p, A; \; 1-p, C] \sim [p, B; 1-p, C]$

Monotonicity

$A \succ B \Rightarrow$
$(p \geq q \Leftrightarrow [p, A; \; 1-p, B] \succeq [q, A; \; 1-q, B])$

Maximum expected utility principle

- Given preferences satisfying the axioms, there exists a utility function $U(s)$ such that

$$U(A) \geq U(B) \Leftrightarrow A \succeq B$$

$$U([p_1, S_1; \ldots; p_n, S_n]) = \sum_i p_i U(S_i)$$

(Ramsey 1931, von Neumann & Morgenstern 1944)

Utilities and humans

- We can try to **elicit** human preferences by presenting humans with lotteries
    - Most of the studies concluded that humans are not rational
        - Many psychologists had a field day with this!
    - Some of it might be that we have limited computing power
    - But some of it would be also be that we do not restrict our thinking at the specific setting of the problem (i.e. only two choices with no other implications, no repeated games and no temporal setting)

Examples of utility calculations in human affairs

- Micromorts: 10^-6 chance of death
  - How much are you willing to pay for a 7th airbag in your car?
- QALY: quality adjusted life-years, useful for medical decisions
  - Who gets the heart transplant etc.

Money

- You can calculate an expected monetary value EVM of a transaction by calculating the expectation of probabilities
- But money does **not** behave as a utility function
- Most people are **risk averse**
  - A decrease in money by X triggers a greater utility change than the same increase
- When deep in dept, people are **risk prone**

Insurance

- How much are you willing to buy this lottery: $[0.5, \$10000; 0.5, 0]$
    - i.e. the certainty equivalent.
- The difference between the certainty equivalent and the EVM is the **insurance premium**
- Why does this work out for the insurance company?
    - They have a different utility curve (more rational)
    - They average over a different lotteries.

Utilities and building agents and robots

- Note that we can build a perfectly rational agent by behavior specification, without ever representing utilities.
- Historically, it had been difficult to build agents by utility specification
    - But this is changing, as we are moving towards more ML and less hardcoded behaviors
- How do you specify the utilities for a self-driving car?
    - Traffic rules?
    - Optimize time to goal, energy consumption?
    - Safety?

Utilities and AGI

- What should be the utilities of an artificial general intelligence?
- **Alignment problem** aligning the AGI with the goals of humanity
- Couple of issues:
    - **Can** we specify the utility of humans?
    - Who gets to specify it? Likely differs from person to person.
    - Are we happy with the human utility function? Eg. pleasure seeking behavior?
    - Wouldn't we better of just placing limits on actions? Eg. Asimov three laws of robotics.
    - Specification gaming
    - Many others...