<u>COT 3100 Final Exam - Part A (Recitation Topics) - 25 pts (4/25/2024) Solutions</u>

1) (5 pts) Steve and Alia must work together to staple 10,000 packets for a conference. Steve can staple one packet in 5 seconds. When working together, the two of them complete the job in five hours and 12 ½ minutes. How many seconds does it take Alia to staple a single packet?

60/5 = 12 packets/min = 60 min/hr x 12/packet/min = 720 packets/hour for Steve

$$720\left(\frac{pack}{hr}\right)\times 5\frac{12.5}{60}hr = (720\times 5 + 12\times 12.5)pack = 3600 + 150 = 3750\ packets$$

This means that Alia must pack 10000 – 3750 = 6250 packets. Her rate (flipped) is:

$$\frac{312.5\ minutes \times 60sec/min}{6250\ packets} = \frac{312.5\times 6}{625} = \frac{625\times 3}{625} = \boldsymbol{3sec/packet}$$

<u>It takes Alia 3 seconds to staple one packet.</u>

Grading: 2 pts to determine total amount of packets stapled by Steve, 1 pt to determine total number of packets stapled by Alia, 2 pts to convert that to seconds per packet.

2) (5 pts) There exist positive integers x and y such that x < y and 61 – xy = 5x + 6y. Without trial and error, determine the values of x and y. Show your work to prove that you avoided trial and error.

61 – xy = 5x + 6y
xy + 5x + 6y = 61
xy + 5x + 6y + 30 = 61 + 30
(x + 6)(y + 5) = 91

Since x and y are positive integers, we aim to express 91 as a product of two integers, each greater than 5. There's only one way to do this: 7 x 13 = 91. Since x < y, it follows that x + 6 = 7 and y + 5 = 13 and that the values of x and y are **<u>x = 1 and y = 8</u>**, respectively.

Grading: 3 pts to get to factored form, 2 pts to reason out unique solution from there.

3) (5 pts) X and Y are both infinite geometric series with a sum of S. The first term of the series X is 10 and the first term of the series Y is 20. If the second term of the series X is 6, find the two following pieces of information: (a) S, (b) The common ratio of the series Y.

Note that the common ratio for series X must be 6/10 = 3/5 since the first and second terms are given. Now, solve for S:

$$S = \frac{10}{1 - \frac{3}{5}} = \frac{10}{\frac{2}{5}} = 25$$

Once we have S, let r be the common ratio of the second series and we get the equation:

$25 = \frac{20}{1-r} \rightarrow 25(1-r) = 20 \rightarrow 25 - 25r = 20 \rightarrow 25r = 5 \rightarrow r = \frac{1}{5}$

S = 25 common ratio of series Y = $\frac{1}{5}$

Grading: 1 pt set up eqn for S, 1 pt solve, 1 pt set up eqn for r, 2 pts solve

4) (5 pts) Let r and s be the roots of the quadratic equation $x^2 - 8x + 11 = 0$. Determine the quadratic equation with leading coefficient 1 which has the roots r^2 and s^2.

Using the given equation, we have r + s = 8 and rs = 11. Squaring the first equation we get:

$r^2 + 2rs + s^2 = 64$
$r^2 + 2(11) + s^2 = 64$
$r^2 + s^2 = 42$

Squaring the second equation we get $(rs)^2 = 11^2$, so $r^2s^2 = 121$.

It follows that the desired quadratic equation is $\mathbf{x^2 - 42x + 121 = 0}$**.**

Grading: 1 pt for writing r and s equations
2 pts for solving for $r^2 + s^2$
1 pt for solving for r^2s^2.
1 pt for writing down the corresponding quadratic

5) (5 pts) Find the values of x and y which satisfy the following system of equations:

$$log_2 x + log_4(8y) = 10$$

$$log_4(4x) + log_2(2y) = 10$$

$$log_2 x + log_4 8 + log_4 y = 10$$
$$log_4 4 + log_4 x + log_2 2 + log_2 y = 10$$

$$log_2 x + \frac{3}{2} + \frac{log_2 y}{log_2 4} = 10$$

$$1 + \frac{log_2 x}{log_2 4} + 1 + log_2 y = 10$$

$$log_2 x + \frac{log_2 y}{2} = 8.5$$

$$\frac{log_2 x}{2} + log_2 y = 8$$

Multiply both of these equations through by 2:

$2log_2 x + log_2 y = 17$ $\qquad$ $log_2 x + 2log_2 y = 16$

Adding these two equations and dividing by 3 yields: $log_2 x + log_2 y = 11$.

Subtract this new equation from the two previous equations to get $log_2 x = 6$ and $log_2 y = 5$.

It follows that **x = 2^6 = 64 and y = 2^5 = 32.**

x = 64 y = 32

Grading: 1 pt equation set up simplifying constant logs
3 pts for solving system for log_2x and log_2y.
1 pt for substituting for both x and y.

Scratch Page – Please clearly mark any work on this page you would like graded.