

**COP 3502 Section 2 Exam #1 - Part 1 (Logic) - 25 pts**

**Date: 9/17/2020**

**Start Time: 1:30 pm EST**

**End Time: 1:55 pm EST**

1) (8 pts) Complete the following truth table. Please write T for true and F for false. It's preferred that you type into the grid given. If you make your own grid, please keep the order of the rows the same and make each entry legible. (Note: There is a not symbol above the xor'ed term.)

| $p$ | $q$ | $r$ | $\overline{p \oplus r}$ | $q \wedge \bar{r}$ | $\overline{(p \oplus r)} \vee (q \wedge \bar{r})$ |
|-----|-----|-----|-------------------------|--------------------|---------------------------------------------------|
| F   | F   | F   |                         |                    |                                                   |
| F   | F   | T   |                         |                    |                                                   |
| F   | T   | F   |                         |                    |                                                   |
| F   | T   | T   |                         |                    |                                                   |
| T   | F   | F   |                         |                    |                                                   |
| T   | F   | T   |                         |                    |                                                   |
| T   | T   | F   |                         |                    |                                                   |
| T   | T   | T   |                         |                    |                                                   |

2) (7 pts) Using the laws of logic, show that the following logical expression is a tautology:

$$p \vee (q \rightarrow r) \vee (\bar{p} \wedge (q \vee \bar{p}))$$

Please use the format shown in class to express your answer.

3) (5 pts) Prove or disprove the following assertion over the universe of integers:

$$\exists x \forall y [xy = 0]$$

Please clearly note whether the assertion is true or not, followed a justification of your answer. Most of the points are awarded for the justification.

4) (5 pts) Prove or disprove the following assertion over the universe of real numbers:

$$\forall x \exists y [xy = 1]$$

Please clearly note whether the assertion is true or not, followed a justification of your answer. Most of the points are awarded for the justification.

**COP 3502 Section 2 Exam #1 - Part 2 (Sets) - 25 pts**

**Date: 9/17/2020**

**Start Time: 1:55 pm EST**

**End Time: 2:20 pm EST**

1) (8 pts) Prove or disprove the following assertion for all sets A, B and C:

$$\text{if } A \subseteq \bar{C}, \text{ then } B - (A - C) \subseteq B - (C - A)$$

2) (8 pts) Prove or disprove the following assertion for all sets A and B:

$$\text{if } \wp(A) \subseteq \wp(B), \text{ then } A \subseteq B$$

3) (9 pts) Let A, B and C be sets that satisfy the following equations dealing with set cardinalities:

$$\begin{aligned} |A \cap B \cap C| &= 4 \\ |A| + |B| + |C| &= 59 \\ |B \cup C| &= 33 \end{aligned}$$

Using this information, determine  $|A \cup (B \cap C)|$ .

**COP 3502 Section 2 Exam #1 - Part 3 (Recitation Material) - 25 pts**

**Date: 9/17/2020**

**Start Time: 2:20 pm EST**

**End Time: 2:45 pm EST**

1) (8 pts) Let  $r$  and  $s$  be the roots of the equation  $x^2 - 8x + 3 = 0$ . What is the quadratic equation with leading coefficient of 1 with the roots  $\frac{r}{s}$  and  $\frac{s}{r}$ ?

2) (8 pts) Dave starts running in a straight line at a constant rate of 20 feet per second. Two minutes later, Ayesha starts from the same spot Dave started at, attempting to catch up with him, running at 25 feet per second. How many **minutes** after Ayesha starts running will she catch up with Dave?

3) (8 pts) Find the ordered pair  $(a, b)$  which satisfies the following pair of equations?

$$2\log_9 a^3 + 3\log_{27} b^2 = 18$$

$$5\log_{27} a^2 + 4\log_9 b^3 = \frac{94}{3}$$

4) (1 pt) A unicorn company is a privately held start up company valued at \$1 billion or more. How many horns does the mythical creature with the same name have?