Group 5 Lab 20(R11)

Group Roles

- Problem Scout
- Explainer
- Explainer
- Question Collector
- Notetaker

Meeting 1

's apartment - 9/28/2019 @ 12:00 PM - (Approx. 1 hr)

Summary:

Group members started our session by discussing Exam 1 materials and expected performance. The last problem on Exam #1 was discussed because of the inclusion of the break that "Jessica" took. Then we broke down how Question 3 Exam #1 should be solved because a majority of the class did poorly on it. The group also briefly went over how the Venn Diagrams look like for Question 4 from Exam #1. After that we went over our selected list of problems. For the first problem we worked on problem 3 from the first quiz to practice our utilization of all the laws of logic and rules of inference in multiple scenarios since we remembered it from the Exam. Problem 6 and 7 from the second recitation was also worked on to further improve our knowledge in using laws of logic. After that we worked on problem 7 from the Exam 1 review just to review knowledge of sets. It was decided to review some problems from the exam so we started with Question 5 from Exam 1 was worked on to make sure that the team understood the inclusion-exclusion principle. The final problems that was worked on was a division question from problem 3 Exam 1. The session ended with the next time and date for the meeting set up.

Outline:

- Group members arrived at the specified meeting location around noon.
- Before diving into the questions, group members discussed their individual experiences concerning Exam 1.
- Shortly after, we began discussing the first question for the session.

3) (9 pts) Using the following given propositions and the rules of inference, prove the conclusion below the dotted line. Note: You may not use all the slots given to you below.

$$(p \vee q) \rightarrow (s \vee t)$$
$$t \rightarrow (u \wedge v)$$
$$s \rightarrow p$$
$$q \vee r$$
$$\bar{r}$$
$$\bar{p}$$

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Question #1 (#3 from Quiz 1)

- We started off by discussing individual solutions to problem 1.
- Jayson had trouble with the question, failing to recognize the application of Modus Ponens.
- Otherwise, there was little discrepancy among answers and approaches.
- Once all members were satisfied and understood the problem, we proceeded to question 2.

Logic Problems

6) Use a truth table to prove the following logical equivalence of the two following Boolean expressions:

(a) $p \rightarrow (q \vee r)$ (b) $\bar{r} \rightarrow (p \rightarrow q)$

7) Use the Laws of Logic to show the equivalence of the two Boolean expressions from question 6.

Question #2 and #3 (#6 and #7 from Recitation #2)

- All members understood and successfully completed question number 2 with no variability between approaches.
- Question 3, much like question 1, resulted in similar approaches from 4 of the members.
- failed to apply the implication identity after applying the commutative law.
- Once all members were satisfied and understood the problem, we proceeded to question 4.

7) (10 pts) Prove or disprove the following assertion about finite sets A, B, and C taken from the positive integers:

$$(A - C) - (B - C) \subseteq (A - B)$$

Question #4 (Exam 1 Review #7)

- This was the problem with the least variety among member submissions.
- All solutions incorporated use of set difference and Modus Tollens.
- Once all members were satisfied and understood the problem, we proceeded to question 5.

4) (10 pts) A school offers three clubs: Art Club, Debate Club and Chess Club. There are 22 students who are either in Art Club or Debate Club and there are 25 students who are either in Debate Club or Chess Club. However, there are no students in both Art and Chess Club. If there are 16 students who are in Debate Club, how many students are in at least one of the three clubs? In order to receive full credit, you must utilize the Inclusion-Exclusion Principle for two and three sets formally. You may draw Venn Diagrams to assist you in coming up with your solution and such a diagram may earn a small amount of partial credit, but most of the credit is earmarked for formally using the Inclusion-Exclusion Principle.

Question #5 (#4 from test 1)

- Members discussed the framework of the Inclusion-Exclusion Principle.
- understood the problem conceptually but had difficulty incorporating the principle.
- Otherwise, all group members arrived at the same answer using the same methodology.
- Once all members were satisfied and understood the problem, we proceeded to question 6.

3) (10 pts) Let n be a positive integer such that 10 | (n - 9). Prove that 8 | (n^2 - 1). In your proof, you may use the result that for all integers a, a(a+1) is an even integer.

Question #6 (#3 from test 1)

- Most group members arrive at the same answer using similar methods.
- Sean had some minor algebraic errors which were easily corrected.
- did not finish the problem, owing to the fact that he did not make the connection that a(a+1) is even for all numbers.
- Otherwise, the problem posed little issue for most members.
- Once all members were satisfied and understood the problem, we discussed a rough date and location for the next meeting, to be ironed out over Discord.
- The meeting was adjourned.

Next meeting time is determined to be 12:00 PM, 10/12/2019 at Tower 3, with any questions or changes to be discussed in the Discord messaging system.

Meeting 2

Tower 3 - 10/12/2019 @ 12:00 PM - (Approx. 1.25 hrs)

Summary:

The group started out by going over problems from the Recitation #4 worksheet. Discussing number #7, we all used the method from class to find the remainder for large numbers n wasn't too sure how to approach question #10 from the worksheet. We went over the division algorithm from class and were able to go through the problem a couple times over to make sure all of us fully understood how to approach similar problems. The group then moved onto questions that were picked personally. en could not break down the process to prove question, so the group went over the notes to explain the steps to solve the problem. We continued to go over the problems found by n. After that, n took us back to the recitation warm up problems because the warmup questions may be on future exams or quizzes. We went over the first problem and had varied approaches, but ended up coming to a proper conclusion. The group meeting concluded after planning out our next meeting.

<u>Outline</u>

- Group members arrived at the chosen location around noon.
- After a short extraneous conversation, group got down to question 1.

7) Without the aid of a calculator, find the value of the remainders for each of the following divisions:

i) $a = 46^4$, $b = 17$ iii) $a = 67^{10}$, $b = 20$
ii) $a = 10103^4$, $b = 10$ iv) $a = 1079^{14}$, $b = 98$

Question 1(#7 From Recitation 10/3/2019)

- All members solved the first question with the same modulus method.
- Once members were satisfied with their approach, members moved on to the next question.

10) In base r, 1000 - 340 equals 440. What is r?

Question 2(#9 From Recitation 10/3/2019)

- The group had mostly similar methods to solve the second question.
- Sean had specific issues, as he forgot what the division algorithm was and how to apply it.
- After the process was explained to him, the group proceeded to question 3 .

35*. Prove that there exist infinitely many pairs of positive integers x, y such that

$$x(x+1) \mid y(y+1), \quad x \nmid y, \quad x+1 \nmid y, \quad x \nmid y+1, \quad x+1 \nmid y+1,$$

and find the least such pair.

Question 3

- n had significant difficulties with this problem.

- Both could not find that x(x+l) = 2·3(12k+5) (18k+7) = 6y, which held them back from understanding the rest of the problem.
- Significant discussion was held on the problem to explain its solution.
- Although a basic understanding of the problem was reached, a note on its relative difficulty was made for later practice.
- After this, the members moved on to question 4.

39. Prove that if a, b, c are any integers, and n is an integer > 3, then there exists an integer k such that none of the numbers $k+a$, $k+b$, $k+c$ is divisible by n.

Question 4

- The group was mostly able to complete this question, however some stumbled at the exact identification of integer k.
- While each was able to find that there must be a remainder r that was different from each individual remainder, some struggled at showing the proof to that form.
- After the process was deliberated between members, with Taiason taking the lead on explanation, each was able to form a complete proof of the problem, the group moved on to question 5.

1) What is the smallest possible positive integer x such that $1260x = N^3$, where N is some positive integer?

Question 5(#1 From Recitation 10/3/2019)

- All members came to the correct solution to the warm up problem using the prime factors of 1260, with slightly differing visualizations of that process.
- After a brief discussion and comparison of these methods, the group decided that each member had an adequate understanding of the process.
- A brief discussion of the next meeting time and location is had, and the meeting was finally adjourned.

Next meeting time is determined to be 12:00 PM, 10/20/2019 at Engineering 2 Atrium, with any questions or changes to be discussed in the Discord messaging system.

Meeting 3

Engineering 2 Atrium - 10/20/2019 @ 12:00 PM - (103 minutes)

Summary

The meeting began with a short discussion about test 2 speculation. We took a focus on studying number theory and induction, with some additional review of previous material. It became apparent to the group that questions that conform to the same general format as those discussed in class are significantly easier than new formats, indicating an issue with respect to overall conceptual understanding and application. Problems from Quiz 2 were examined due to their relevance to number theory and average quiz performance from group members. The harmonics problem from homework 5 proved to be confusing for most group members, so we thought it apt to bring up for discussion. While most group members felt they had a generally adequate grasp of number theory, it was almost unanimous among members that induction was a sore spot that required more individual review before the upcoming exam. Once all the selected problems were discussed, we set up a time for the next meeting and concluded the session.

Outline

1) (15 pts)

(a) (10 pts) Find one ordered pair of integers (x, y) that satisfies the equation $137x + 49y = 1$.

(b) (3 pts) Using your work from part (a), determine 49^{-1} mod 137. Please give an answer in between 0 and 136, inclusive.

(c) (2 pts) Using your work from part(a), list the set of all ordered pairs of integers (x, y) that Satisfy the equation $137x + 49y = 1$.

Please clearly mark each of your final answers with a box around each answer, and clearly indicate which of the three parts each answer is for.

Question 1(#1 From Quiz 2)

- Many members were able to solve part a easily.
- ı struggled with understanding the reasoning behind part b but otherwise did not have problems with the rest.
- , however, struggled with understanding and solving part c.
- Discussion helped them and the whole group understanding mods and number theory better.

5. (2000 AMC 12 9) Mrs. Walter gave an exam in a mathematics class of five students. She entered the scores in random order into a spreadsheet, which recalculated the class average after each score was entered. Mrs. Walter noticed that after each score was entered, the average was always an integer. The scores (listed in ascending order) were 71,76,80,82,and 91. What was the last score Mrs. Walter entered.

Question 2 (math.cmu.edu)

- This question was pretty difficult for the whole group.
- Most of the issues stemmed from not knowing how to start the problem.
- After discussion and trial and error, the group managed to get to the solution together.
- More discussion afterwards helped each member clarify that they understand the theory behind the problem.

12. When 30! is computed, it ends in 7 zeros. Find the digit that immediately precedes these zeros.

Question 3 (isinj.com)

- This was a fairly difficult problem for the whole group.
- Many members knew how to start the problem but didn't get the right answer.
- got the right answer and explained their method and the reasoning behind the question.
- The group agreed that they all have adequate understanding of the solution but that questions that do not conform to the structure of those discussed in class pose significant issues for all team members.

2) (10 pts) Let a be an integer such that $a \equiv 3 \pmod 8$. Prove that $a^2 \equiv 9 \pmod{16}$. (Hint: Use the mod equation to express a in terms of another integer, and then use this when substituting for a^2, reducing the expression using the rules of mod, under mod 16.)

Question 4(#2 From Quiz 2)

- Half of the group struggled to solve this problem while the other half understood it enough to solve.
- A group discussion helped everyone understand the substitutions needed to solve the problem.
- Everyone agreed that they understand the solution well enough.

3) For all positive integers n, define the n^{th} Harmonic number, H_n, as follows: $H_n = \sum_{i=1}^{n} \frac{1}{i}$. For all positive integer, using induction on n, prove that $\sum_{i=1}^{n} H_i = (n+1)H_n - n$.

Question 5 (#3 from Homework 5)

- This problem was added as some members struggled with understanding how to solve it.
- reviewed the group on induction.
- Most of the struggling came from not understanding harmonic numbers and how would one properly substitute for the Inductive Hypothesis.
- After discussion members agreed that they better understood the problem.

Next meeting time is determined to be 12:00 PM, 10/26/2019 at Engineering 2 Atrium, with any questions or changes to be discussed in the Discord messaging system.