Group Section: 20
Group Number: 3

Group Members:

COT3100 Group Write-Up 1

Group Meetings:

1. Date: 9/18/2019 Time: 3:00pm-5:00pm Location: John C. Hitt Library
2. Date: 10/4/2019 Time:4:00pm-6:00pm Location: John C. Hitt Library
3. Date: 10/18/2019 Time:4:00pm-6:00pm Location: John C. Hitt Library
4. Date: 10/ 22 /2019 Time:1:20pm-3:00pm Location: John C. Hitt Library

Meeting 1: 9/18/19

Outline:

- Started the meeting by sharing what each member wanted to discuss in preparation for the first quiz
    - Took those ideas and chose those that were more challenging or a topic brought up by multiple members of the group
        - Rules of Inference and certain algebra questions were a consensus
    - Assigned roles for future meetings:
        - Notetaker
        - Problem Scout:
        - Explainer
        - Question Collector:
- Went over recitation problems that we did not finish completing
    - Quickly reviewed the questions the group had completed in the lab as a refresher:
        - Question: 2
            - In a recent basketball game, Shenille attempted only three-point shots and two-point shots. She was successful in 20% of her three-point shots and 30% of her two-point shots. Shenille attempted 30 shots. How many points did she score? Is it possible to determine how many two-point shots Shenille attempted? If the percentage of each type of shot she made was different, would it be possible to determine how many points she scored?
        - Question: 5
            - In base 10, the number 2013 ends in the digit 3. In base 9, on the other hand, the same number is written as 26769 and ends in the digit 6. For how many positive integers b does the base b representation of 2013 end in the digit 3?
    - Proceeded with laws of logic questions and shared strategies for how to approach them.
        - Question: 8
            - Using the following premises:

              $(\overline{p} \vee \overline{q}) \rightarrow (r \wedge s)$

              $r \rightarrow t$

$\neg t$
Derive the conclusion p.

        - Question 9:
            - In class, Modus Ponens was proved using just the laws of logic. Prove the Rule of Resolution in the same manner. Namely, show that the following is a tautology via the laws of logic. $((p \lor q) \land (\overline{p} \lor r)) \to (q \lor r)$

- Began studying for Exam 1 by looking at past exam questions in the archive and choosing topics that group members felt they needed more practice or clarification (such as general algebra, universal generalization, set proofs)
    - Question 4:
        - For an open statement P(x, y), it is known that $\exists x[\forall y(P(x, y)]$. Is it necessarily true that $\forall y[\exists x(P(x, y)]$? If it is the case, prove it, if the assertion is false, create a specific open statement P(x, y) for which $\exists x[\forall y(P(x, y)]$ is true and $\forall y[\exists x(P(x, y)]$ is false and explain why the first is true and the second is false.
    - Problem #2 from class notes: 9/17/19
    - Practice Exam: Question 9:
        - Let $S = \{2x\text{^ }2 + x - 6 = 0 | x \in Z\}$. (Note: Recall that Z is the set of integers.) Explicitly list each element that belongs to the set S. Put a circle around your final answer.

Summary:

Throughout the duration of our first study session, we were able to come together as a group and build each other's knowledge. We will summarize specific problems that were difficult for us , and how we were able to overcome this gap in knowledge.

While reviewing the recitation problems we talked about question number two that is listed above in this week meeting outline.As a group we thought that we would have to compute how many of each shot that Shenille had taken. After some trial and failure we realized that was not needed. We could instead use x as the number of two point shots and 30-x to represent her three point shots. From there we can use the percentages of completion of shots with our variables and in the end, the variable x canceled out and just left us with the total points. That means her total shots could have been represented as (3/10*x)*2+(2/10)*(30-x)*3 = total points. This would then give us the answer 18.

When handling laws of logic problems, many of us struggled. Up to this point in our mathematical careers, we were able to approach problems as if there was only one way to arrive at an answer using pre-designed steps. With these logic problems, we had to change the way we thought of mathematics. We really decided that we needed to focus on how each step interacted with the step before. To aid in this we decided that we should first look logic in a more general application. We each searched logic warmup problems and found a site that offered free practice problems.
https://brilliant.org/practice/logic-puzzles/?subtopic=puzzles&chapter=puzzles
The problems varied in difficulty level from simple questions like :
Kevin, Joseph, and Nicholas are 3 brothers. If the following statements are all true, which of them is the youngest?

- Kevin is the oldest.
- Nicholas is not the oldest.
- Joseph is not the youngest.

We also attempted a more difficult question :

Cornwallis, Geoffrey, and Markett are outside the Pearly Gates discussing theology. One of them is an angel who always tells the truth, one of them is a demon who always lies, and one of them is a spirit who can either tell the truth or tell a lie.

1. Cornwallis says: "I am a spirit."
2. Geoffrey says: "Cornwallis is a demon."
3. Markett says: "Geoffrey is an angel."
4. Who is the spirit?

The aim of these somewhat trivial problems, way to stimulate the thought process that we would have to undergo when handling logic laws in this class. After this initial step was completed, we went ahead and familiarize ourselves with the logic laws so that when we attempted or chosen problems we were more comfortable with them. We then practiced problems 8 and 9 that related to this topic. For question 9, we reviewed the example in class to ensure we had a grasp on how to prove the Rule of Resolution using the logic laws. We each tried the question individually and then shared our results and cleared up any confusion other members had.

It was important that before we started dealing with nested quantifiers, we did a recap of what each symbol meant. We started with simple statements to practice how we would represent these meanings using quantifiers. We took statements such as: Some person is the oldest person in the world, and using our quantifiers and sets converted them to the mathematical representation. Seeing the application in real-life circumstances helped a lot of us when we did question #2 and question #4.

In relation to nested quantifiers, used in Question 4, as a group we struggled with certain word usage. We were not sure about the difference between there exists a y for all x and for all x there exists a y. When there exists a y for all x is used this means that for any natural number x, there is some number y that satisfies the statement for all x and makes it true. When talking about for all x there exists a y this means for every x there is at least one y value that will satisfy the statement and make it true.

Problem 9 from the class notes was an interesting problem because it combined solving for the root and sets. Some of our group struggled with the factorization because the highest degree x had a coefficient of 2 so old algebra strategies had to be reviewed with how to factor it effectively. After some revision with factoring by grouping we were able to get (2x-3)(x+2)=0. x=3/2 or x=-2. We noticed that 3/2 was not an integer so the only thing that was left to be contained on the set would be {-2}.

Interesting:

What we found interesting is that we had to do a lot of foundational work before we even began these problems. It was not as simple as picking out a problem and then practicing it. We found it very interesting how logical thinking using the practice of real-world problems was able to help us solve logic law problems and how our initial ideas for how to complete a problem weren't always the most effective (such as with Question 2). We also found Question 9 to be interesting because many of the set problems we had seen didn't involve any algebra so the combination of first doing algebra and then using the result in our set answer was very different from the types of problems we had practiced, such as doing set proofs.

Meeting 2: 10/4/2019

Outline:

- Started the meeting off by studying the results from Quiz 1
    - Narrowed down topics to discuss based on group members results on the quiz and any questions they may have for clarification
    - Had Angel and Conner bring up any unresolved issues from the last meeting to discuss for this meeting
    - Quiz questions reviewed:
        - Question 2:
            - Let r1 and r2 be the roots of the quadratic equation x^2 + 9x + 5 = 0. What is the value of $r1/r2 + r2/r1$?
        - Question 3:
            - Using the following given propositions and the rules of inference, prove the conclusion below the dotted line. Note: You may not use all the slots given to you below.
              $(p \lor q) \rightarrow (s \lor t)$
              $t \rightarrow (u \land v)$
              $s \rightarrow p$
              $q \lor r$
              $\overline{r}$
              $\overline{p}$
              ----
              u
- Went over recitation problems (#4) that we did not finish completing
    - Many group members did not remember what arithmetic or geometric progression was so that the topic was researched and reviewed:
        - Question 5:
            - The sequence log12 162, log12 x, log12 y, log12 z, , log12 1250 is an arithmetic progression. What is x?
    - All of us were confused about Question 1 and tried finding resources to approach it when Angel discovered a resource about prime factorization.
        - The group then read over what prime factorization was and how to use it and then attempted to figure out question 1.
        - Question 1:

        - What is the smallest possible positive integer x such that 1260x = N3 , where N is some positive integer?
    - All members of our group felt comfortable with quotients and remainders of small numbers and converting values to Base 10 so we moved on to remainders of large numbers based on the suggestion of Chris.
        - Question 7:
            - Without the aid of a calculator, find the value of the remainders for each of the following divisions:
              i) a = 464 , b = 17 iii) a = 6710, b = 20
              ii) a = 101034 , b = 10 iv) a = 107914, b = 98
        - Then moved on to practice more mod questions chosen by Chris.
        - Question 9:
            - Prove the following: Let the remainder when a is divided by b be r, where a, b > 0 and 0 ≤ r < b. Show that a^2 ≡ r^2 (mod b), formally. That is, prove that b | (a^2 - r^ 2).
- Reviewed for quiz 2
    - All group members wanted to review mod and extended Euclidean
    - Chose to review questions from class on 10/3/2019
        - Question:
            - For given integers (positive) a,b,c find all integer solutions to the equation 216x+87y = 3.
        - Question:
            - If an is an integer such that a ≡ 5(mod 12), prove a^2 ≡ 1 mod 24.

Summary:

At the start of this meeting, we all were able to review the quiz and see where we made mistakes. The members in our group that performed better were able to show how they approached each question and what strategies helped them to be successful. After this review, we targeted problems that we were unable to master in our last meeting. In our general algebra review, we realized that collectively we were not strong using arithmetic and geometric progression, so we each watched a short youtube video in order to ensure we were on the same page. One member from our group, Chris, was strong when it came to logarithmic computation, so he helped us to review when handling question #5.

Within our group, there was a misconception of inference rules. We did not realize that half of the statements after the arrow could not be replaced with something else. To help with this we went back over each rule and wrote rules down on a whiteboard. We then watched a youtube video on rules of inference and supplemented this with previous years' problems from the archive. We were able to pinpoint where we went wrong and then took preventative measures to ensure that this mistake was not made on the exam. We realized that everyone was comfortable with quotients and remainders and in order to allocate time wisely we moved on to other material. Chris, the problem scout, came prepared with a few problems that would summarize well what we had learned in class during the week. Problem 7 was fairly simple as we had gained a strong base in mods and remainder. Problem 9 that included a divisibility proof did prove more difficult. We struggled because we were not working with concrete numbers , but variables. We were able to resolve this problem by finding a similar proof from our class notes that we were able to follow step by step and then apply that same thinking to our problem.

After the problems collected by Chris were complete, we then started revisions for quiz 2. As a group, we decided that we needed more practice with the Euclidean algorithm. We individually reviewed our class notes and then watched a youtube video and came back together to ask questions we still needed clarified. The main issue we discovered was being careful when substituting the equations into the others and not causing any algebra errors. After this, we practiced additional problems from class(10/3/2019).
The question that dealt with integer solutions was something that required lots of group collaboration. Allison explained that the best approach was to factor out the three on each side so that the numbers were more manageable. After this the substitution was very straight -forward, but the simplification while plugging everything in was harder to organize.

Interesting:

Something that we found was interesting was the different ways that modulo and remainder problems can be asked, when the process for solving is relatively similar. We were surprised to see that we often struggled with this operator so we went back to the notes to fully understand this operator and how it differed from programming which we are all familiar with. We had also discussed the similarity of a previous recitation problem in comparison to quiz problem number 2. It surprised us that you can manipulate the values of r1 and r2 in different manners without actually solving the quadratic. During our progression of solving inference problems, we noticed the similarity yet difference between it and logic problems. The implication symbol often would restrict our manipulation of the statements whereas in logic many statements were interchangeable which we often found interesting to deal with. In regards to the Euclidean, at first glance it seemed like many of the numbers had come from nowhere because for most of us this was our first real encounter with it, but after some analysis we realized how each subsequent statement was related to the other that came before it by the remainder and the term that was dividing.

Meeting 3: 10/18/2019

Outline:

- Started the meeting off by going over quiz 2 results.
  - Decided to work through both problems again as a group as a refresher
    - Question 1:
      - (a) (10 pts) Find one ordered pair of integers (x, y) that satisfies the equation 137x + 49y = 1. (b) (3 pts) Using your work from part (a), determine 49-1 mod 137. Please give an answer in between 0 and 136, inclusive. (c) (2 pts) Using your work from part(a), list the set of all ordered pairs of integers (x, y) that Satisfy the equation 137x + 49 y = 1.
    - Question 2:
      - Let a be an integer such that a ≡ 3 (mod 8). Prove that a2 ≡ 9 (mod 16). (Hint: Use the mod equation to express an in terms of another integer, and then use this when substituting for a2 , reducing the expression using the rules of mod, under mod 16.)
- Went over recitation problems (#5) that we did not finish completing
  - All members felt they needed extra practice with inequality induction problems as well as harmonic.
  - Discussed the difference between a regular induction problem and a strong induction problem.
  - Reviewed algebra problems as a refresher before diving into induction.
    - Question 1:

            - Five positive consecutive integers starting with a have average b. What is the average of 5 consecutive integers that start with b, in terms of a?
        - Question 3:
            - If f(n+1) = (2f(n+1))/2, and f(1)=2, what is f(101)?
    - Chris, the problem scout, suggested we go over Questions 7 and 8 from lab.
        - Question 6:
            - The nth harmonic number, denoted Hn, is defined as follows: $H_n = \sum_{i=1}^{n} 1/i$. Prove that the following equation is true for all positive integers n, using induction on n: $\sum_{i=1}^{n} i/(i+1) = (n+1) - H_{n+1}$
        - Question 7:
            - Use induction on n to prove the following inequality for all positive integers n:
            - $\sum_{i=0}^{n} 3^i < \frac{3^{n+1}}{2}$
- Reviewed distance problem from exam 1 in preparation for exam 2
    - We discussed how it was mentioned in class that many people forgot to include the 15-minute break listed in the problem
    - Discussed strategies for how to approach distance problems and some misconceptions that could lead us astray
    - Question 8:
        - Jessica is going from Orlando to Miami and she takes a 15-minute break at Vero Beach. Her goal is to average 60 miles per hour for the whole trip. The distance between Orlando and Vero Beach is 100 miles and the distance between Vero Beach and Miami is 140 miles. If her average driving speed from Orlando to Vero Beach is 50 miles per hour, how fast must her average speed be driving from Vero Beach to Miami to achieve her goal? (Note: I don't actually advise driving this fast ever!!!)
- Reviewed and discussed other exam problems members had issues with:
    - Question 6:
        - Prove or Disprove: For arbitrary sets, A, B, C, if $A \cap B \cap C = \varnothing$, then $(A \subseteq B) \vee (A \subseteq C)$
    - Question 3:

        - Let n be a positive integer such that $10 \mid (n - 9)$. Prove that $8 \mid$ (n2 - 1). In your proof, you may use the result that for all integers a, a(a+1) is an even integer.
- Assigned each member a portion of the group report to complete and have prepared for the last meeting before the exam

Summary:

As we began to review the quiz questions we noticed that some of us had made similar errors in computing the vast amount of algebra involved in the first problem. More specifically, the extended Euclidean had posed difficulty throughout the distribution step and addition that follows afterwards. After, comparing our solutions we easily identified some group members simple mistakes as well as strategies to double check our solutions.

Q8: A major problem some members faced in answering the question, was not taking in consideration of the 15 minute break. The 15 minute break definitely took a toll on the average speed of the driver on their way to Miami. The correct way to answer the question was to get the total time from Orlando to Vero Beach, add the 15 minute break, then calculate the total and subtract the prior times to figure out the time it takes to get to Miami.

Q3: We discussed 2 ways to approach the 3rd question in the first exam. The first way we discussed that with the given info 10|(n-9), n had to odd. With that info we plugged n in to 8|(n^2-1). 8|4(c+1) = 2|c+1, divisible by 2 making it true. And this way compared to the posted solution, this explanation was more clear and concise for some group members and offered insight in to other ways to solve the same problem.

Q6: The majority of the group could not come up with ways to clearly construct a counterexample, hence failing the question on the exam. As a result the team took extra time to review and come up with a counterexample and strategies. Firstly learning the actual definition and the meaning of the symbols for sets and how to apply them correctly allowed us to see where we first went wrong for the exam question. We were so focused on making sure the sets had nothing in common that we failed to realize some elements could overlap in two sets as long as all three did not contain a similar element. That was the key to coming up with the correct counterexample for the test but making that error in a way helped us learn to recognize and be more careful when analyzing set questions.

Q6 on recitation: Upon breaking the summation, one error that we came across was that we simplified the right side to be the induction hypothesis plus the value of (k+1) plugged into the harmonic equation when instead it should be the induction hypothesis plus (k+1)/(k+2) . This showed us that we have to be very careful when substituting in equations and make sure we know what is equivalent.

Interesting:

One thing the group found interesting during this particular meeting was by the 15 minutes break differed the answer in a significant way. Since we had experienced some trouble with it how to account for it in the equation, we found it more surprising following our conclusion of the answer. Another interesting thing we found was the difficulty it took to find a counterexample to number 6. In class, the counterexample problems seemed relatively simple; however, this was not the case when it came time to do it ourselves. It definitely was a shock to find out that we needed much more practice with this type of problem. Lastly, when looking at example harmonic equations, we were interested to find how to split up the harmonic equation so that we ended with only an H_{k+1} and not H_{k+1} and H_k using the relation $H_{k+1} = H_k + \frac{1}{k+1}$.

Meeting 4:10/22/2019

Outline:

- Short meeting to finalize report.
  - Group worked together using Google Docs
  - Made suggestions to add more details while keeping the report succinct

Summary:

The last meeting each member arrived with their completed parts of the group report that were assigned at the previous meeting. Allison created a Google Doc and shared it so that each member could fill in their info for the report and combine all the information to make one complete and succinct report. We checked over each other's work to ensure that the ideas and topics discussed in all the meetings were included and represented in enough detail. Ro-Hanna, the notetaker of the group, verified that the written report matched her notes from each group meeting and if something was accidentally left out or forgotten, she could remind the member who worked on that portion to add to it. Chris, the Problem Scout, made sure that each member included all the questions he chose for each meeting in their report and explained why these questions were chosen (either as a refresher or to stimulate discussion about topics that some members needed clarification on).