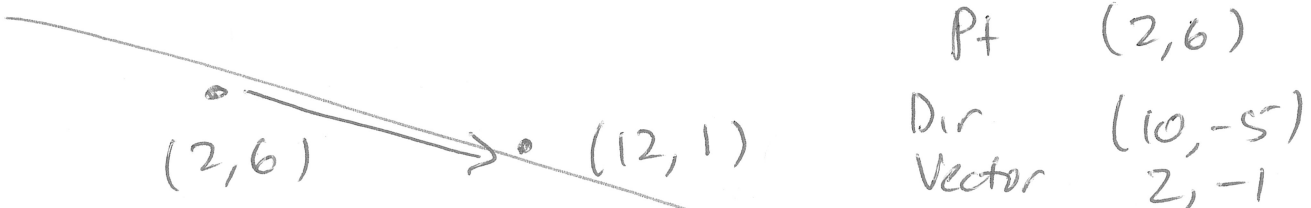
COP4516 2/24/26 Geometry 2D

1) Coordinate Geometry
   - Point
   - Lines → Vectors
   - Line Segments
     - Dot Product
     - Cross Product
   - Intersections
   - Distances
   - Convex Hull

2) School Geometry w/ shapes
   - triangles
   - polygons
   - circles

Points - DIST FORMULA
USEFUL SQUARE OF DISTANCE
(avoid floating pt)

Line - Defined by 2 pts



Pt (2,6)

Dir Vector (10,-5) 2,-1

Vector Eqn ← $r = (2,6) + \lambda(10,-5)$

$x = 2 + 10\lambda$
$y = 6 + (-5)\lambda$
} parametric

$$x = -3 + \beta$$
$$y = -9 + 4\beta$$

$$\left.\begin{aligned} 2 + 10\lambda &= -3 + \beta \\ 6 - 5\lambda &= -9 + 4\beta \end{aligned}\right\}$$

Cramers Rule

$$\begin{aligned} 10\lambda - \beta &= -5 \\ -5\lambda - 4\beta &= -15 \end{aligned}$$

$$den = \begin{vmatrix} 10 & -1 \\ -5 & -4 \end{vmatrix} = 10(-4) - (-1)(-5) = -40 - 5 = -45$$

$$num\lambda = \begin{vmatrix} -5 & -1 \\ -15 & -4 \end{vmatrix}$$

$$= (-5)(-4) - (-15)(-1)$$

$$= 20 - 15 = 5$$

$$\lambda = \frac{5}{-45} = -\frac{1}{9}$$

$$num\beta = \begin{vmatrix} 10 & -5 \\ -5 & -15 \end{vmatrix}$$

$$= 10(-15) - (-5)(-5)$$

$$= -150 - 25$$

$$= -175$$

$$\beta = \frac{-175}{-45} = \frac{35}{9}$$

$$2 + 10\left(-\tfrac{1}{9}\right),\ 6 - 5\left(-\tfrac{1}{9}\right)$$

$$\left(\frac{8}{9}, 6\frac{5}{9}\right)$$

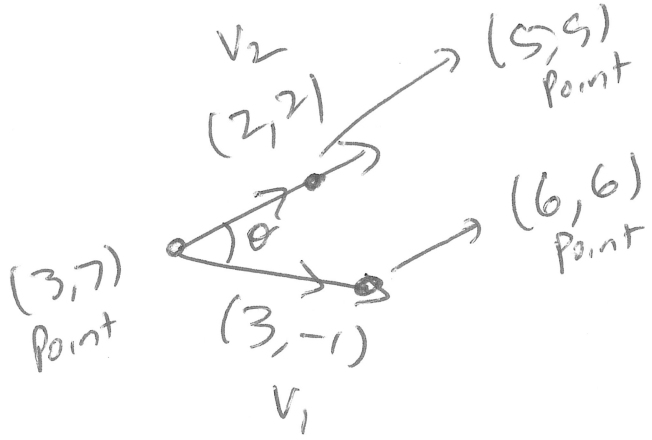
* Line Segment

$$0 \le \lambda \le 1$$

$$0 \le \beta \le 1$$

~~test~~

```
if (lambda < -EPS ||
    lambda > 1+EPS)
    return null;
```



$4 = \sqrt{8}\sqrt{10}\cos\theta$

$V_1 \cdot V_2 = |V_1||V_2|\cos\theta$

$V_1 = (2,2)$

$V_2 = (3,-1)$

$6 - 2 = 4$

$\cos\theta = \frac{4}{\sqrt{80}} = \frac{4}{4\sqrt{5}} = \frac{1}{\sqrt{5}}$

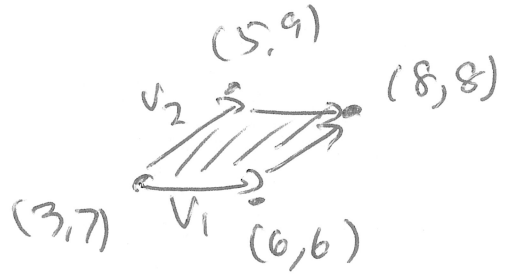
$a\cos(\quad)$

$V_1 = a_1\vec{i} + b_1\vec{j}$

$V_2 = a_2\vec{i} + b_2\vec{j}$

$a_1a_2 + b_1b_2$

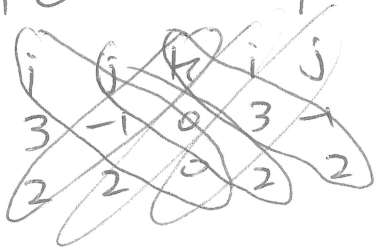
$|V_1| = \sqrt{a_1^2 + b_1^2}$



Area of this is Magnitude of the cross product of V_1 and V_2

$V_1 \times V_2$

$$\begin{vmatrix} i & j & k \\ 3 & -1 & 0 \\ 2 & 2 & 0 \end{vmatrix} = 8 = 3\times 2 - (2)(-1) =$$



$= 0i + 0j + 6k - (-2)k - 0i - 0j$

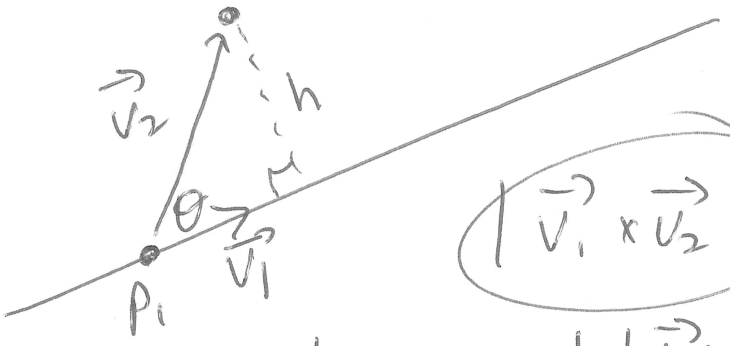
$8k$

$= |V_1||V_2|\sin\theta$

$$\text{fn area} = \frac{1}{2}bc\sin A$$

↑ included angle

Line to Point Distance



$$\left|\vec{v}_1 \times \vec{v}_2\right| = |v_1|\,|v_2| \sin\theta$$

$$\frac{1}{2}\left|\vec{v}_1 \times \vec{v}_2\right| = \frac{1}{2}|v_1||v_2|\sin\theta$$

$$= \text{tri area}$$

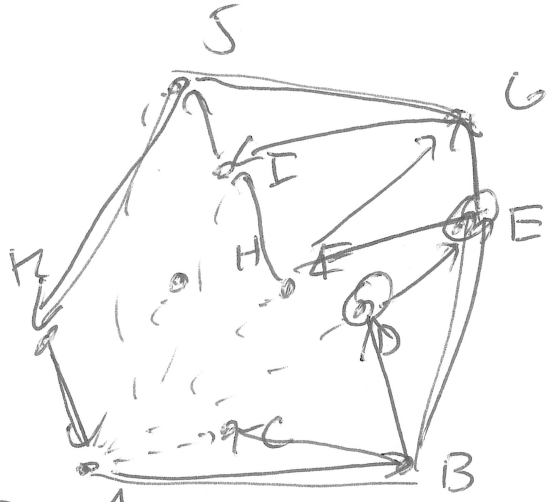
$$\sin\theta = \frac{h}{|v_2|}$$

$$\text{tri area} = \frac{1}{2}|v_1|h$$

$$\frac{1}{2}|v_1|h = \frac{1}{2}|v_1||v_2|\sin\theta$$

$$h = \frac{\left|\vec{v}_1 \times \vec{v}_2\right|}{|v_1|}$$

Convex Hull + Poly Area



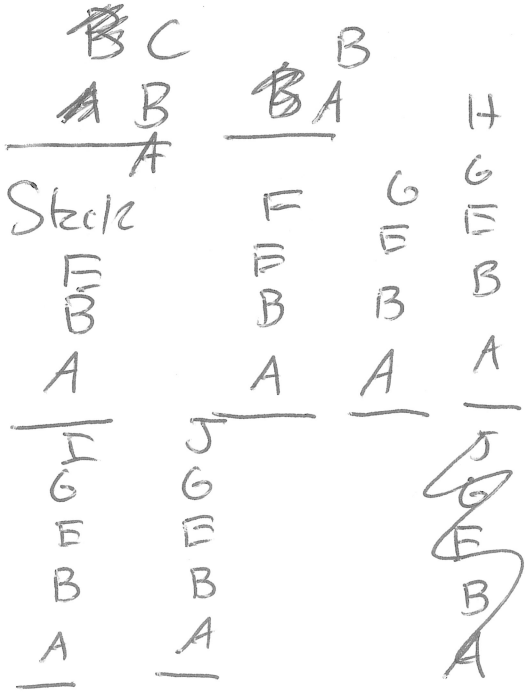
Graham Scan
also
Top + Bottom Hull

(1) Bottom left A

(2) Sort pts by angle from bot left (0 and π) $[0, \pi)$

(3) Push 1st 2 onto stack

(4) Loop through pts
consider the last turn
BCD
keep on popping items off stack until you get a right turn



Polygon $(x[0], y[0])$
$(x[1], y[1])$
$(x[2], y[2])$
⋮
$(x[n-1], y[n-1])$

```
int doubleArea = 0;
for (int i=0; i<n; i++)
    doubleArea += (x[i] * y[(i+1)%n] -
                   x[(i+1)%n] * y[i]);
return abs(doubleArea)
```

only @ end

neg

subtracted out

Law of Sines
Law of ~~Cos~~ Cosines
} triangle

Circle - Line Intersection