Coexistence in Heterogeneous Spectrum Through Distributed Correlated Equilibrium in Cognitive Radio Networks

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Abstract

Coexistence protocols enable collocated cognitive radio networks (CRNs) to share the spectrum in an opportunistic manner. These protocols work under the assumption that all spectrum bands provide the same level of throughput. This assumption is however limited in scope because channel conditions as well as the licensee's usage of allocated channels can vary significantly with time and space. Under these circumstances, CRNs are expected to have a preference over the choice of available channels which can lead to an imbalance in contention for disparate channels, degraded quality of service, and an overall inefficient utilization of spectrum resource. In this paper, we analyze this situation from a game theoretic perspective and model the coexistence of CRNs with heterogeneous spectrum as a non-cooperative, repeated spectrum sharing game. We derive three solutions for the game; 1) pure and 2) mixed strategy Nash Equilibria as well as 3) centralized and distributed correlated equilibria which are derived using linear programming and a channel selection learning algorithms, respectively. We also analyze each of these solutions from the perspective of fairness and efficiency. To that end, we utilize the concept of *price of anarchy* to measure the efficiency of these solutions under selfish behavior from CRNs.

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Preprint submitted to Elsevier *October 11, 2015*

Keywords: Cognitive Radio, Coexistence, Game Theory, Correlated Equilibrium, Price of Anarchy.

1. Introduction

The TV white space (TVWS) channels in the 54-698 MHz frequency range have been made available by the Federal Communications Commission (FCC) [1] for secondary unlicensed access. This is because of a realization that the gap between the demand and supply of wireless spectrum resource is ever increasing and fixed spectrum allocation is causing its severe under-utilization [2]. Strict requirements are placed on the Secondary Users (SU) of the spectrum which is otherwise allocated to licensees called primary users (PU), to continuously sense the spectrum and vacate it when the presence of the PU is detected and not to cause them any interference. This type of spectrum access is intuitively called Dynamic Spectrum Access (DSA). Cognitive Radio Network (CRN) is a paradigm that meets precisely this communication criterion and utilizes DSA to enable secondary, unlicensed access to TVWS spectrum bands in an opportunistic and non-interfering basis [1].

DSA allows CRNs to ensure that their use of spectrum does not cause interference to PUs while at the same time all spectrum opportunities are utilized to the maximum. The decision to select a specific channel for DSA is usually made by a central entity in the CRN such as its base station or some algorithm that enables all SUs in the CRN to reach a consensus in a distributed manner. IEEE 802.22 wireless regional area network (WRANs) [3] is an example of a CRN in which the base station controls all the operation including the choice of spectrum bands for communication. Regardless of how a decision to utilize a specific channel is made, every entity in the CRN is bound to abide by that decision. However, reaching a consensus is non-trivial in the case of multiple collocated CRNs in a given region, all of whom compete for access to the same set of available channels. This situation is called self co-existence in the context of CRNs which employ coexistence protocols to deal with such situations.

1.1. Problem Definition

Most coexistence protocols work under the assumption that all spectrum bands afford the same level of throughput and do not take into consideration the fact that these channels can be heterogeneous. The heterogeneity of channels can be in the sense that they may vary in their characteristics such as signal to noise ratio (SNR) or bandwidth. Furthermore, a channel whose PU remains idle for most of the time may be more attractive to a CRN as compared with a channel with high PU spectrum usage. This would entail that channels can have an associated *quality* parameter and CRNs may have a preference over the set of available channels for secondary access. Without any incentive for altruism, all CRNs would want to gain access to the highest quality channels making it a conflict condition. Therefore, *in the absence of any mechanism to enforce fairness in accessing varying quality channels, ensuring coexistence with fair spectrum allocation and efficient spectrum utilization for CRNs is likely to become a very difficult task.*

Game theory provides an elegant means to model strategic interaction between agents which may or may not be cooperative in nature. It has been applied to numerous areas of research involving conflict, competition and cooperation in multi-agent systems which also encompass wireless communications. Therefore, by leveraging the mechanisms of game theory, we model the heterogeneous spectrum sharing in CRNs as a repeated, non-cooperative anti-coordination game in which collocated CRNs in a given region are its players, as shown in figure 1. The payoff for every player in the game is determined by the quality of the spectrum band to which it is able to gain access.

In a preliminary version of this paper [15], we presented a study of various game theoretic solutions for the problem of self coexistence in the context of heterogeneous spectrum resources. In this paper, we provide a detailed discussion on the problem and its proposed solution. We also present a detailed mathematical analysis on fairness and efficiency of the solution through the concept of *Price of Anarchy* which is an analysis tool that measures a system's degradation in the presence of selfish behavior from its entities. We also confirm our findings

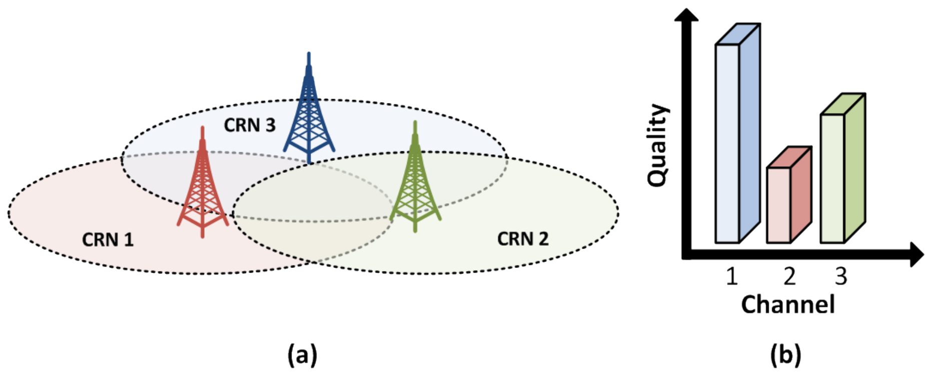

Figure 1: (a) Collocated CRNs competing for (b) Heterogeneous channels. The channels of the spectrum band may vary in quality with respect to availability, bandwidth or SNR, etc.

through detailed simulations.

1.2. Contribution

In this paper, we have formulated a heterogeneous spectrum sharing anti-coordination game to come up with a solution that results in fair and efficient utilization of the spectrum resources. Specifically, we have made the following contributions:

- As potential solutions for the heterogeneous spectrum sharing game, we have derived the game's pure and mixed strategy Nash Equilibria (PSNE and MSNE respectively) as well as its Correlated equilibrium (CE).
- We have analyzed the game's solutions in the context of fairness and efficiency and demonstrated that the traditional solution concepts of Nash Equilibria (NE) are either inefficient or unfair. We also show that the strategies in CE are optimal as well as fair while sharing heterogeneous spectrum resource.
- Finally, to show that CE is scalable, we have demonstrated how CE can be achieved in a 2-player as well as an N-player game with centralized

as well as a distributed approach using linear optimization and channel selection learning algorithm, respectively.

2. Related Work

In this section we provide an overview of some of the works carried out in the domain of self coexistence in CRNs as well as application of the game theoretic solution concept of correlated equilibrium in the context of communication networks.

A game theoretic approach based on correlated equilibrium has been proposed in [4] for multi-tier decentralized interference mitigation in two-tier cellular systems. Authors of [5] propose a multi-cell resource allocation game for efficient allocation of resources in orthogonal frequency division multiple access (OFDMA) systems based on throughput, inter-cell interference and complexity. The subcarriers are considered as players of the game while the base station acts as the provider of external recommendation signal needed for achieving correlation of strategies of players.

The solution concept of Nash equilibrium has been adopted for distributed spectrum management, relay selection and queuing in [6, 7] for interference-limited cooperative wireless networks. The authors have proposed a distributed best-response algorithm to develop a Branch and Bound-based algorithm solving the associated social problem. Authors of [8] model the competition among multiple femtocell base stations for spectrum resource allocation in an OFDMA LTE downlink system as a static non-cooperative game. The correlated equilibrium of the game is derived through a distributed resource block access algorithm which is a variant of the No-Regret learning algorithm. CRNs with SUs having variable traffic characteristics are considered in [9] to tackle the problem of distributed spectrum sensing by modeling it as a cooperative spectrum sensing game for utility maximization. The authors have proposed another variant of the no-regret learning algorithm called neighborhood learning (NBL) which achieves correlated equilibrium for the spectrum sensing game. In contrast to the no-regret learning algorithm, NBL is not completely distributed and requires some coordination among players to achieve better performance.

Correlated equilibrium has been employed in [10] for a P2P file sharing non-cooperative game to jointly optimize players expected delays in downloading files. Not uploading files for others causes an increase in file download time for all players which in turn, forces even the non-cooperative players to cooperate. The authors of [11] tackle the self-coexistence problem of finding a mechanism that achieves a minimum number of wasted time slots for every collocated CRN to find an empty spectrum band for communications. To do so, they employ a distributed modified minority game under incomplete information assumption.

Different punishment strategies have been employed in [12] that form part of a Gaussian interference game in a one-shot game as well as an infinite horizon repeated game to enforce cooperation. Spectrum sharing is however considered within the context of a single CRN. Evolutionary game theory is applied in [13] to solve the problem in a joint context of spectrum sensing and sharing within a single CRN. Multiple SUs are assumed to be competing for unlicensed access to a single channel. SUs are considered to have half-duplex devices so they cannot sense and access a channel simultaneously.

Utility graph coloring is used to address the problem of self-coexistence in CRNs in [14]. Allocation of spectrum for multiple overlapping CRNs is done using graph coloring in order to minimize interference and maximize spectrum utilization using a combination of aggregation, fragmentation of channel carriers, broadcast messages and contention resolution. The authors of [16] achieve correlated equilibrium with the help of No-regret learning algorithm to address the problem of network congestion when a number of SUs within a single CRN contend for access to channels using a CSMA type MAC protocol. They model interactions of SUs within the CRN as a prisoners dilemma game in which payoffs for the players are based on aggressive or non-aggressive transmission strategies after gaining access to idle channels.

3. System Model and Assumptions

3.1. System Model

As shown in figure 1, we consider a region where IEEE 802.22 WRAN based CRNs represented by the set of $\mathcal{N} = \{1, 2, ..., n\}$ players are collocated and contend for secondary access to the licensed spectrum bands. The set of TVWS channels available for secondary access by the contending CRNs is represented as $\mathcal{K} = \{1, 2, ..., k\}$ channels. The spectrum consists of channels that differ from each other due to various network parameters such as noise, bandwidth or even availability. These differences make the spectrum heterogeneous in nature with channels considered to have some 'quality' parameter determined by the payoff that a CRN may achieve if it is able to gain access to that channel. The notations commonly used in this paper are shown in table 1.

3.2. Assumptions

Following are the underlying assumptions for the work presented in this paper:

- ***Time:*** A single MAC superframe constitutes one time slot. Every CRN needs to gain access to a channel for which it contends with all other collocated CRNs in every time slot. One superframe's time slot is also treated as one iteration in the spectrum sharing game.

- ***Spectrum opportunity and wastage:*** A given time slot's spectrum opportunity arises due to a PU being idle in its allocated channel. The opportunity may result in a collision and be wasted if two or more CRNs select the same channel for accessing in the same time slot.

- ***Knowledge about PU activity:*** In addition to the FCC mandated continuous spectrum sensing to detect PUs' activity, CRNs are also required to periodically access online databases such as [17, 18] in order to gain up-to-date information about licensed PUs operating in a given region.

- ***Channel quality:*** The amount of PU activity, bandwidth and SNR which, for the purpose of this paper collectively determine a channel's quality can be learnt from online databases and measured through spectrum sensing over a period of time. Due to the fact that all contending CRNs are collocated in a given region, it is reasonable to assume that a given channel's quality is common knowledge.

- ***History of channel access:*** As stated above, all CRNs are collocated in a given region and are contending for the same spectrum resource. Coexistence Beacon Protocol (CBP) of the IEEE 802.22 standard [3] specifies how collocated CRNs are *required* to exchange information about their operations including their current operating channels. Therefore, every CRN can tell which channels other CRNs were able to gain access to in previous time slots and maintain their channel access history.

- ***Non-cooperative behavior:*** All CRNs are rational about their choices and want to maximize their *own* payoffs only. This means that at the start of every time slot, every player has a clear preference of selecting the best available channel without sharing. Consequently, if every player tries to access the best channel without a central mechanism to resolve contention, it will result in a collision and the spectrum opportunity being wasted because of the non-cooperative behavior.

- ***Payoffs*[2]*:*** Players[3] that eventually gain access to higher quality channels will gain higher payoffs as compared to the players that end up with lower quality channels. In the subsequent section, we show that our proposed spectrum sharing game can be implemented solely on the basis of a CRN's own payoff observations.

[2]We use the terms utility and payoff interchangeably.

[3]Similarly, we use the terms CRNs and players interchangeably

4. Equilibrium Solutions for Heterogeneous Spectrum Sharing Game

In this section, we first present the formulation of our proposed spectrum sharing game, followed by the derivation of pure and mixed strategy NE. Next we introduce the concept of CE and demonstrate how it can be achieved in a centralized implementation for a 2-player game using linear optimization. We also demonstrate that CE can be achieved in a distributed manner for an N-player game using a learning algorithm called channel selection learning algorithm which is an adaptation of the No-Regret (NR) learning algorithm [19]. Using these concepts we model the problem of self-coexistence and heterogeneous spectrum sharing in the following subsections as an anti-coordination game framework. The game is a non-cooperative repeated game with perfect information because:

- Being rational players, CRNs compete for the best channels available in the spectrum band and are interested only in maximizing their own utility. Therefore, CRNs are not bound to cooperate with each other.
- Utilities are common knowledge since the quality of various network parameters can be measured by every CRN. Also, every CRN can tell which channels other CRNs were able to gain access to in the past hence they know other CRNs' payoffs.

4.1. Game Formulation

The heterogeneous spectrum sharing anti-coordination game presented in this paper is represented as $\mathcal{G} = \langle \mathcal{N}, (\mathcal{A}), (\mathcal{U}) \rangle$. Players in the game $\mathcal{G}$ are CRNs represented by $\mathcal{N}$. Every player in the game has the same action space represented by $\mathcal{A} = \{a_1, a_2, ..., a_k\}$ and the set of utilities of the channels is $\mathcal{U} = \{u_1, u_2, ..., u_k\}$. Let $\mathcal{K} = \{1, 2, ..., k\}$ denote the set of available channels and there is a bijection between the sets $\mathcal{A}$ and $\mathcal{K}$. Also, Let N and K represent the total number of collocated CRNs and the total number of available channels, respectively. Strategy a_k means selecting channel k for communication and a player gets a payoff of u_k if he selected channel k and no other player selected the same channel for a given time slot. The payoff for players playing strategies a_k and a_j when competing against each other is denoted by the ordered pair $u(a_k, a_j) \in \mathcal{U}$ and is a function of an individual channel's quality given by:

$$u(a_k, a_j) = \begin{cases} (u_k, u_j) \;\; when \; k \neq j \\ (0, 0) \;\; when \; k = j \end{cases} \qquad u_k, u_j > 0 \tag{1}$$

where the first element of the ordered pair $u(a_k, a_j)$ represents the payoff for player that selected channel k and the second element for player that selected channel j. For the sake of clarity and ease in analysis and without any loss of generality, we assume that $u_k > u_j, \forall u \in \mathbb{R}^k_{\geq 0}$. Initially, we consider a game with 2 players and 2 heterogeneous channels. Later, we present the case with N-players and K-channels in section IV-D. The game represented by 1 can also be represented in strategic form as table 2, which shows the payoffs for two players selecting channels k or j. Since $u_k > u_j$, it is in every CRN's interest to choose channel k instead of channel j for a larger payoff. However, when the players select the same channel it results in a collision, the spectrum opportunity being wasted and both player end up with a payoff of 0. On the other hand, if both players select different channels then their payoffs reflect the quality of the channel to which they are able to gain access. As shown in table 2, this game is the reverse of the classic *Battle of the Sexes* game and is classified as an anti-coordination game where it is in both players' interest not to end up selecting the same strategy.

4.2. Pure and Mixed Strategy Nash Equilibria for the Spectrum Sharing Game

In this subsection we derive the solution concepts in the form of pure strategy Nash equilibria (PSNE) as well as the mixed strategy Nash equilibrium (MSNE) for our spectrum sharing anti-coordination game.

Definition 1: The *Pure Strategy Nash Equilibrium* [20] of the spectrum sharing game is an action profile $a^* \in \mathcal{A}$ of actions, such that:

$$u(a_i^*, a_{-i}^*) \succeq u(a_i, a_{-i}^*), \forall i \in \mathcal{N} \tag{2}$$

where $\succeq$ is a preference relation over payoffs of strategies a_i^* and a_i. The above definition means that for a_i^* to be a pure strategy NE, it must satisfy the condition that no player i has another strategy that yields a higher payoff than the one for playing a_i^* given that every other player plays their equilibrium strategy a_{-i}^*.

Lemma 1: Strategy pairs (a_k, a_j) and (a_j, a_k) are pure strategy NE of the anti-coordination game.

Proof: Assume player 1 to be the row player and player 2 to be the column player in table 2. From equation (1) it follows that both u_k and u_j are positive values and therefore the payoffs for strategy pairs (a_k, a_j) and (a_j, a_k) are greater than the payoffs for strategy pairs (a_k, a_k) and (a_j, a_j). Consider the payoff for strategy pair (a_k, a_j) from table 2. Given that the player playing strategy a_j continues to play this strategy, then from definition 1 for NE, it follows that the player playing strategy a_k does not have any incentive to change his choice to a_j i.e., it will receive a smaller payoff of 0 if it switched to a_j. Therefore, (a_k, a_j) is a PSNE. The same argument can be applied to prove that the strategy pair (a_j, a_k) is the second PSNE of this game. ■

Definition 2: The *Mixed Strategy Nash Equilibrium* [20] of the spectrum sharing game is a probability distribution $\hat{p}$ over the set of actions A for any player such that:

$$\hat{p} = (p_1, p_2, ..., p_{|\mathcal{K}|}) \in \mathbb{R}_{\geq 0}^{|\mathcal{K}|}, \text{and} \sum_{j=1}^{|\mathcal{K}|} p_j = 1 \tag{3}$$

which makes the opponents indifferent about the choice of their strategies by making the payoffs from all of their strategies equal. Let α be the probability with which player 1 plays strategy a_k and $\beta = (1 - \alpha)$ be the probability of playing strategy a_j, then from the payoffs of table 2, the expected utility of player 2 for playing strategy a_k is given by:

$$EU_2(a_k) = \alpha u(a_k, a_k) + \beta u(a_j, a_k) = \alpha.0 + \beta u_k \tag{4}$$

Similarly, the expected utility of player 2 for playing strategy a_j is given by:

$$EU_2(a_j) = \alpha u(a_k, a_j) + \beta u(a_j, a_j) = \alpha u_j + \beta.0 \tag{5}$$

According to definition 2, player 2 will be indifferent about the choice of strategies when the expected utilities from playing strategies a_k and a_j are equal, i.e.,

$$EU_2(a_k) = EU_2(a_j) \tag{6}$$

Substituting (4) and (5) in (6), we have $\beta(u_k) = \alpha(u_j)$. Therefore:

$$\alpha = \frac{u_k}{u_k + u_j} \tag{7}$$

$$\beta = 1 - \alpha = \frac{u_j}{u_k + u_j} \tag{8}$$

The mixed strategy NE for the heterogeneous spectrum sharing game is given by the distribution $\hat{p} = \{\alpha, \beta\}$ of equations (7) and (8) which means that when both players select strategies a_k and a_j with probabilities α and β respectively, then their opponents will be indifferent about the outcomes of the play. To generalize, expected utility for every player i in a K-channel heterogeneous spectrum sharing game is given as follows:

$$EU_i = \sum_{m=1}^{|\mathcal{K}|} u_m.p_m, \forall m \in \mathcal{K} \tag{9}$$

where p_m represents the probability of CRN i selecting channel m all other CRNs not selecting channel m. We will utilize eq (9) in section V for the fairness and efficiency analysis of the various game equilibria.

4.3. Centralized Correlated Equilibrium for 2-Player Game

Under pure and mixed strategy NE, it is assumed that the players choose their strategies independently and without any prior coordination. However as we demonstrate next, it is in every player's interest to coordinate their actions such that the outcomes are favorable to all players by avoiding the selection of same channel. Players would maximize their utilities if somehow they could avoid ending up selecting the same channels. A coordination, or the lack thereof, in selecting channels would essentially make it an *anti-coordination* game. Such a coordination to avoid selecting same channels can be achieved with the help of a mutually trusted central entity that can provide all players with a recommendation signal. The external recommendation signals can either be public or private signals or they can even be learnt over a period of time eliminating the need for a central entity making possible its distributed implementation. In this subsection, we present the centralized algorithm to achieve the centralized correlated equilibrium (CE) for a 2-player, 2-channel game while the distributed algorithm to achieve CE with a channel selection learning algorithm for an N-player K-channel game is presented in the next subsection.

CE is a state in which, when given the availability of an external recommendation signal, none of the players can achieve a greater utility by ignoring that signal when all other players follow the recommended action. In other words, π is a correlated equilibrium if no strategy modification can result in an increase in a player's expected utility. Formally, CE is defined as:

Definition 3: A probability distribution π is a *Correlated Equilibrium* of a game when [21]:

$$\sum_{a_i \in \mathcal{A}} \pi(a_i, a_{-i})[u_i(a_i, a_{-i}) - u_i(a_i', a_{-i})] \geq 0, \forall i \in \mathcal{N} \tag{10}$$

$\pi(a_i, a_{-i})$ is the joint probability distribution of players to select a certain strategy pair in the next time slot. The inequality (10) represents that selecting some different strategy a_i' instead of a_i in the next time slot will not result in a higher payoff for a player given that all other players adhere to the recommended strategy. In a centralized implementation of correlated equilibrium for a 2-player 2-strategy game such as the one shown in table 3, any external entity e.g., one of the contending CRNs may be selected as the recommender that calculates and provides the external recommendation signal for all contending CRNs according to the CE joint probability distribution $\pi = (p_{1,1}, p_{1,2}, p_{2,1}, p_{2,2})$. The strategic form of such a correlated strategy pair is shown in table 3. A correlated strategy pair means that the action pair (a_1, a_1) is played with probability $p_{1,1}$ and action pair (a_1, a_2) is played with probability $p_{1,2}$ etc.

Here we derive the centralized CE of the heterogeneous spectrum sharing game using a linear optimization approach. CE can be implemented for a multi-player game using linear optimization; however, with this method the number of constraints for CE grows exponentially with the number of players and their strategies and the problem grows at a polynomial rate [22]. Therefore, we derive centralized correlated equilibrium only for a 2-player game and consider the case for an N-player game in the next subsection when we present the case for a decentralized CE. Let the objective function J to find the optimal strategy CE for a 2-player game be defined as:

$$J = \max_{p_{i,j}} \sum_{i=1}^{2} \sum_{j=1}^{2} [u_1(a_i, a_j) + u_2(a_i, a_j)] p_{i,j} \tag{11}$$

where the constraints for CE in equation (11) are:

$$p_{1,1} + p_{1,2} + p_{2,1} + p_{2,2} = 1 \tag{12}$$

$$p_{1,1}u_1(a_1, a_1) + p_{1,2}u_1(a_1, a_2) \geq p_{1,1}u_1(a_2, a_1) + p_{1,2}u_1(a_2, a_2) \tag{13}$$

$$p_{2,1}u_1(a_2, a_1) + p_{2,2}u_1(a_2, a_2) \geq p_{2,1}u_1(a_1, a_1) + p_{2,2}u_1(a_1, a_2) \tag{14}$$

$$p_{1,1}u_2(a_1, a_1) + p_{2,1}u_2(a_2, a_1) \geq p_{1,1}u_2(a_1, a_2) + p_{2,1}u_2(a_2, a_2) \tag{15}$$

$$p_{1,2}u_2(a_1, a_2) + p_{2,2}u_2(a_2, a_2) \geq p_{2,2}u_2(a_1, a_1) + p_{2,2}u_2(a_2, a_1) \tag{16}$$

$$p_{1,2} = p_{2,1} \text{ and } p_{1,1} = p_{2,2} = 0 \tag{17}$$

For the game of table 2, any correlated equilibrium of the form $\pi = (0, p, 1-p, 0)$ such as the one represented in table 3 will maximize the sum of expected payoffs for the players because it eliminates the possibility of the players contending for the same channel. The constraints needed for a CE represented by equations 13 to 16 mean that given a recommendation signal is received, the players do not have an incentive to unilaterally deviate from the recommended strategy. This can be explained as follows: Consider the constraint of equation 13. It suggests that player 1 has been recommended channel 1 for use during current stage of the game. Since the recommendation signal is not binding for the two players, player 1 does not exactly know player 2's chosen channel. Then, the expected payoff from choosing the recommended strategy (channel 1) will yield *at least* the same utility as switching to another strategy (choosing channel 2 instead of the recommended channel 1). Therefore, player 1 does not have an incentive to unilaterally switch from the recommendation signal. The same explanation also holds for constraints of equations 14 to 16 when the two players are recommended available channels under the other possible scenarios. For an egalitarian equilibrium which is fair and maximizes the sum of expected payoffs, we have an additional constraint defined as (17).

Having the recommender to provide external signal based on equation (11) and the constraints (12) to (17), ensures that probability of the two players ending up in the same channel is minimized so that the likelihood of spectrum opportunity being wasted is also minimized and hence players' utilities can be maximized. It must be noted that the external recommendation signal is not binding and players are free to ignore recommended actions. The efficiency of avoiding the collision condition is achieved only because the players know that they will achieve higher payoffs by following the recommendation signal. This argument is explained with the help of following example.

Consider a situation in which the recommender selects an egalitarian CE probability distribution $\pi = (0, 1/2, 1/2, 0)$ over the payoff matrix of table 3 in order for the two players to avoid selecting the strategy pairs (a_1, a_1) and (a_2, a_2). Suppose the external signal randomly recommends player 1 to select action a_2 i.e., channel 2 which is of lower quality and results in a smaller payoff of 7 compared with a payoff of 9 if channel 1 was selected for next time slot. Player 1 knows that player 2 will follow the recommended action because it has been recommended a higher quality channel. It is however in player 1's interest to select the action recommended by the external signal since it would yield a higher payoff of 7 instead of 0 if external signal is ignored and both players end up selecting the same higher quality channel.

4.4. Distributed Correlated Equilibrium for N-Player Game

CE for a 2-player game was derived in the previous subsection and in this subsection we consider the case for an N-player K-channel game and demonstrate how CE can be achieved in a distributed manner and without the need of any communication among the CRNs or an external recommendation signal. To this end, we propose a novel channel selection learning algorithm which is an adaptation of the No-Regret learning algorithm [19] to achieve CE. Channel selection learning algorithm is based on the concept of minimizing a CRN's regret in the hindsight for not selecting a particular channel in every time slot up to the current time t. Next we detail the working of the channel selection learning algorithm.

Channel Selection Learning Algorithm: Suppose that the heterogeneous spectrum sharing game $\mathcal{G}$ is played repeatedly at every time slot $t = 1, 2, 3, \cdots$ and every CRN knows the history of plays h_t of every other CRN up to time t because of being collocated. Given a history of play $h_t = (a_i^\tau)_{\tau=1}^t$ up to time t, every CRN calculates a probability $p_i^{t+1} \in \pi(\mathcal{A}_i)$ of selecting the same channel a_i^τ for the next time slot. The probability for selecting a channel for the next time slot is calculated as follows: for every two different channel choices $a'_i \in \mathcal{A}$ and $a_i \in \mathcal{A}$ up to time t, if every CRN replaces channel a_i with channel a'_i every time that it was selected in the past then its utility for time τ

will become:

$$\omega_i^\tau(a_i') = \begin{cases} u_i^\tau(a_i', a_{-i}^\tau) \text{ if } a_i^\tau = a_i \\ u_i^\tau(a_i^\tau) \text{ otherwise} \end{cases} \quad \forall i \in \mathcal{A} \tag{18}$$

Then the average difference in a CRN's payoff up to time t is given by:

$$\delta_i^t(a_i', a_i) = \frac{1}{t}\sum_{\tau=1}^{t}[\omega_i^\tau(a_i') - u_i^\tau(a_i^\tau)], \forall a_i' \neq a_i \tag{19}$$

and CRN i's average regret at time t is given by:

$$NR_i^t(a_i', a_i) = [\delta_i^t(a_i', a_i)]^+, \forall a_i' \neq a_i \tag{20}$$

then the probabilities of selecting channels a_i and a_i' in the next time slot are a function of a CRN's average regret and given by:

$$p_i^{t+1}(a_i') = \frac{1}{\mu} NR_i^t(a_i', a_i) \tag{21}$$

$$p_i^{t+1}(a_i) = 1 - \sum p_i^{t+1}(a_i'), \forall a_i' \neq a_i \tag{22}$$

The parameter μ determines the amount of inertia (or un-willingness) that a CRN possesses in deviating from its current choice of a given channel and its value is constrained by $\mu > 2M_i(|\mathcal{K}| - 1)$, such that $|\mathcal{K}|$ is the number of channels available for contention and M_i is the upper bound on $|u_i(.)|$. Its value is independent of time as well as the play's history and also ensures that there is always a positive probability of staying in the same channel as in the previous time slot. As $t \to \infty$, the empirical probability distribution π over the N-tuples of strategies converges to the CE [22]. A summary of the Channel Selection learning is given in Algorithm 1.

5. Fairness and Efficiency of Derived Equilibria

Having demonstrated how CE can be achieved for an N-player K-channel game, we now provide an analysis on the fairness and efficiency of all of the equilibria derived in this paper. For the sake of clarity and easy analysis we

Algorithm 1: Channel Selection Learning Algorithm

Data: μ, M_i(upper bound on $|u_i(.)|$)

Result: Every channel's probability of being selected by every CRN for the next time slot.

Initialization: $t \leftarrow 1, p_i^1(a_i) = \frac{1}{|\mathcal{K}|}$;

---

while *CRNs contend for heterogeneous channels* **do**

  for *every CRN i* **do**

    Compute current Regret NR_i^t up to time t for not selecting channel $a_i' \in \mathcal{A}$ as per equation (20);

    Calculate p_i^{t+1} i.e., prob. of selecting channel a_i and all other channels a_i' for the next time slot as per equations (21) and (22);

    $t \leftarrow t + 1$;

  end

end

---

consider the case of a 2-player 2-channel heterogeneous spectrum sharing game while the same arguments can be applied for analyzing an N-player K-channel scenario. There are three different types of equilibria computed in preceding subsections for the spectrum sharing heterogeneous game:

- Two pure-strategy NE for the anti-coordination game (a_k, a_j) and (a_j, a_k).
- A mixed strategy NE defined by the probability distribution $\hat{p} = \{\alpha, \beta\}$ given by equations (7) and (8).
- A Correlated Equilibrium defined by the probability distribution $\pi = (0, p, 1-p, 0)$ over joint strategy pairs of table (3) given by equation (11) and constrained by equations (12) to (17).

Price of Anarchy: To analyze the efficiency of these equilibria, we first introduce Price of Anarchy (PoA) [23], a measure of degradation due to selfish behavior of non-cooperating players in a system. Let $S \subseteq \mathcal{A}$ be a set of strategies in equilibrium such that S_P, S_M and S_C refer to the sets of strategies in pure strategy NE, mixed strategy NE, and CE for the heterogeneous spectrum sharing game, respectively. We define the measure of efficiency of the game as a utility function $F : S \to \mathbb{R}$ such that

$$F(a) = \sum_{i=1}^{|\mathcal{N}|} u_i(a) \tag{23}$$

then PoA is defined as the ratio between optimal efficiency and the worst equilibrium efficiency of the game, as follows:

$$PoA = \frac{\arg\max_{a \in \mathcal{A}} F(a)}{\arg\min_{a \in S} F(a)} \tag{24}$$

where the strategies $a \in S$ represent progressively higher efficiency as PoA approaches 1.

Optimal Efficiency: The heterogeneous spectrum sharing game will result in optimum efficiency when all of the contending CRNs always select different channels i.e., they are able to avoid contention for the same channel which would result in a collision and zero payoff. In the presence of selfish players, such optimality is only possible with a correlated choice of strategies as well as fair distribution of spectrum resource. When these conditions are satisfied then the maximum value of the utility function $F(a)$ is given as the sum of utilities of all channels as follows:

$$\arg\max_{a \in \mathcal{A}} F(a) = \sum u_m, \forall m \in \mathcal{K} \tag{25}$$

Next we discuss the fairness of equilibria as well as their efficiency by deriving the worst equilibrium efficiencies and comparing them with optimal efficiency of the game (25).

PoA with Pure Strategy Nash Equilibria: As assumed previously in section 3, channel k is of higher quality than channel j therefore $u_k > u_j$. Then from the payoff matrix of table I, gaining access to channel k brings a larger payoff to a CRN whereas being of comparatively lower quality, channel j brings a smaller payoff. There are two pure-strategy Nash equilibria (a_k, a_j) and (a_j, a_k), however both of them are unfair because $u_k \neq u_j$ and one player always gets a smaller payoff than the other. Since the game is a non-cooperative game and every player is interested in maximizing its own payoff, all of them will end up selecting the larger payoff channels resulting in contention and collision in every time slot and hence zero payoffs. As a result PoA is not defined in the context of PSNE of this game and therefore, PSNE is not a practical solution for this game.

PoA with Mixed Strategy Nash Equilibrium: MSNE of our spectrum sharing game is the probability distribution $\hat{p} = \alpha, \beta$ given by equations (7) and (8). Since the expected utilities EU_i given by equation (9) for all players are equal when they mix their strategies according to the distribution $\hat{p}$, we can conclude that MSNE is fair.

We now derive the PoA for the game to be able to determine its efficiency under MSNE. There is only one MSNE of the game therefore, minimum value of the utility function $F(a)$ under MSNE is the sum of expected utilities of every player from equation (9) and is given by:

$$\arg\min_{a \in S_M} F(a) = |\mathcal{N}|.u_i. \sum_{m=1, m \neq i}^{|\mathcal{K}|} p_m, \forall i \in \mathcal{N} \tag{26}$$

and the price of anarchy under MSNE is given by

$$PoA_M = \frac{\sum_{m=1}^{|\mathcal{K}|} u_m}{|\mathcal{N}|.u_i. \sum_{m=1, m \neq i}^{|\mathcal{K}|} p_m} \tag{27}$$

by substituting equation (23) and (26) in (24).

PoA with Correlated Equilibrium: The correlated equilibrium (CE) of a 2-player 2-channel spectrum sharing game is defined by the probability distribution of tuple $\pi = (0, p, 1-p, 0)$ over joint strategy pairs constrained by equation (18). Equations (18) to (22) represent the channel selection learning algorithm implemented to achieve CE for an N-player K-channel scenario. Correlation in the choice of strategies ensures that the probability of players selecting the same channel for contention is minimized so that the spectrum opportunity is not wasted due to collision and players' payoffs are maximized. As demonstrated in next section, the NR algorithm takes some time to converge to CE during which they may select the same channels resulting in collisions. However after convergence, the contending CRNs *never* select the same channel thus wastage of spectrum opportunities is avoided altogether and all channels are utilized to the maximum. Therefore, minimum value of the utility function $F(a)$ under CE is the sum of expected utilities of every player given as:

$$\arg\min_{a \in S_C} F(a) = \sum_{m=1}^{|\mathcal{K}|} u_m \tag{28}$$

and PoA under CE is given as:

$$PoA_C = \frac{\sum_{m=1}^{|\mathcal{K}|} u_m}{\sum_{m=1}^{|\mathcal{K}|} u_m} = 1 \tag{29}$$

Discussion: Under the constraint for the MSNE probabilities of selecting different channels $0 < p_m < 1$ and $\sum_{m=1}^{|\mathcal{K}|} p_m = 1$, minimum value of the utility function $F(a)$ in equation (26) will always be smaller than 1. This means that PoA under MSNE of equation (27) will always be greater than 1. On the other hand CE has a price of anarchy equal to 1 which according to its definition, is the most efficient case. This is a clear evidence of CE not only being fair but also the most efficient solution for the problem of heterogeneous spectrum sharing game.

Figure 2 illustrates the overall scenario of non-cooperative behavior from CRNs for self coexistence and the improvement that can be achieved with our proposed channel selection learning algorithm. Figure 2a shows how selfish behavior may result in collision and wastage of spectrum resource. figure 2b depicts a scenario where MSNE results in a fair yet inefficient spectrum utilization while figure 2c shows performance improvement achieved through CE.

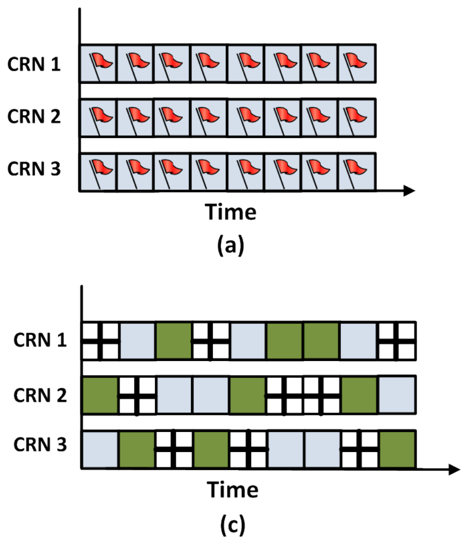

Figure 2: Channel access pattern of CRNs. (a) Selfish behavior from CRNs for best quality channel (channel 1) will always result in a collision. (b) Fair distribution of spectrum resource when CRNs mix their choice of channels according to MSNE. However, MSNE is inefficient becuse of collisions and wasted opportunities. (c) Fair and efficient resource distribution with CE.

6. Simulations and Results

6.1. Simulation Setup

For the purpose of validating the effectiveness of CE, we implemented our proposed anti-coordination game along with the channel selection learning algorithm. We verify that CE is achievable, fair and efficient as it always yields a higher expected utility per CRN as compared with MSNE. For the purpose of simulation, n represents the number of CRNs and k represents the number of channels in the spectrum available for secondary access by the CRNs. We first carry out the comparison of CE and MSNE with a 2-player 2-channel game i.e., $n = 2$ and $k = 2$ and calculate expected utilities per CRN. Later we carry out simulations with varying number of CRNs and channels and demonstrate that the game always converges to CE. Since the channel selection learning algorithm approaches CE based solely on a given network's own payoff observations, it allows the distributed implementation of our proposed anti-coordination game. Inertia parameter of the channel selection learning algorithm is μ whose value is kept constant for all simulations except for the simulation of figure 1 in which we demonstrate the effect of changing the values of μ.

6.2. Simulation Results

Figure 3 shows a comparison of expected utilities per CRN under MSNE and CE with various values for the inertia parameter μ. Payoff value for channel 1 is $u_1 = 9$ while channel 2 has a payoff of $u_2 = 7$. Compared with all the four plots for CE in figure 3 where the expected utilities converge to 8 per CRN, MSNE yields a smaller expected utility of 3.93 per CRN, proving our analysis that CE is more efficient than MSNE. Different values of μ achieve CE at different rates however the convergence values are identical. As evident from figure 3, μ being the inertia parameter, reflects a CRN's propensity towards staying in the same channel in next time as the previous one.

Figure 4 shows a comparison of CE at different values of the number of networks (n) and channels (k). For this simulation, the number of CRNs is kept the same as the number of channels available for contention i.e., $n = k$ such that utilities of the channels are $u_1 = 9$, $u_2 = 7$ and $u_3 = 6$. With every additional CRN, a *lower* quality channel was added to the spectrum resulting in smaller expected utility per CRN and slower convergence to equilibrium.

Figure 5 shows the CE for expected utilities per CRN over time such that $k \geq n$ i.e., increasing the number of available channels from 4 to 6 while keeping the number of contending CRNs constant at 4. Notice that the convergence value for expected utility is the same for all cases. It shows a very important aspect of the channel selection learning algorithm which allows CRNs to always have a fair as well as an efficient distribution of channel resources as players choose the highest quality channels from the pool of available channels. Also, the speed of convergence to CE is fastest when the number of CRNs is equal to the number of available channels i.e., $n = k$. Payoff values for channels 1 through 6 for this simulation are kept at 9, 7, 6, 5, 4 and 3 respectively.

Figure 6 shows the CE for expected utilities per CRN over time such that $n \leq k$ i.e., decreasing the number of CRNs from 4 to 2 while keeping the number of available channels constant at 4. Intuitively, expected utility per CRN is lowest at $n = 4$ and $k = 4$ as compared with the situation when the number of contending networks is smaller however, the speed of convergence to CE is fastest when $n = k$. Payoff values for channels 1 through 4 are 9, 7, 6 and 5 respectively.

Finally, figure 7 shows the results of simulation when $n \geq k$ and the number of channels is kept fixed while the number of contending CRNs is increased. It shows that as soon as the number of CRNs contending for channels becomes more than the number of channels available, there will always be at least one collision between two or more CRNs in every time slot making the expected utility per CRN to drop significantly. However, the channel selection learning algorithm still manages to achieve CE despite much degraded expected utilities per CRN.

Figure 8 shows the effect of varying channel quality on the expected utilitiy for every CRN. When the payoff for channel 2 is dropped from a value of 7 to 5, it results in a proportional drop in the expected utility from 8 (a) to 7 (b). A proportional drop is shown in (c) when channel 2's payoff further drops to 3. Similar results are expected from a greater number of channels and this is shown in figure 9 where the number of channels is 3. In this figure, payoff from channel 3 is reduced progressively with similar results. As can be noticed in figures 8 and 9, another aspect of varying channel quality is that the rate of convergence to CE decreases with the increase in variance of channel quality.

7. Conclusions

Coexistence protocols employed by collocated CRNs usually do not take into consideration the fact that spectrum bands vary significantly with regards to channel quality thereby making some channels of the spectrum bands more attractive to CRNs than others. In this paper, we aimed at solving the problem of sharing heterogeneous spectrum by adopting a game theoretic approach. By analyzing the system's efficiency and fairness with the help of *price of anarchy*, we demonstrated that correlated equilibrium solves the problem of inefficiency and unfairness associated with the game solutions of pure and mixed strategy Nash equilibria. Furthermore, to address the problems associated with a centralized implementation, we proposed the use of a novel channel selection learning algorithm that enables the CRNs to achieve correlated equilibrium in a distributed manner.

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Table 1: Notations & Acronyms

Notation	Definition
a_k	CRN's action of selecting channel k
u_k	CRN's utility for gaining access to channel k
$\mathcal{N}$	set of contending CRNs
$\mathcal{K}$	set of available channels
$\hat{p}$	prob. distr over set of channels (in MSNE)
EU_k	Expected Utility from accessing channel k
π	joint prob. distr of available channels (in CE)
t	current time
$\omega_i^\tau(a_i')$	utility if all a_i in time slot τ were replaced by a_i'
$\delta_i^t(a_i', a_i)$	average difference in utility upto time t if CRN i replaces channel a_i every time that it was selected in the past, with channel a_i', $\forall a_i' \neq a_i$
μ	Inertia or un-willingness of a CRN to change its strategy
$NR_i^t(a_i', a_i)$	CRN i's average regret upto time t for selecting channel a_i instead of every other other channel a_i' that was not selected
p_i^{t+1}	prob. of selecting a channel for next time slot
S	Set of strategies in equilibrium
$F(a)$	utility function for all actions in equilibrium
PoA	Price of Anarchy
PU	Primary User
SU	Secondary User
NE	Nash Equilibrium
PSNE	Pure Strategy Nash Equilibrium
MSNE	Mixed Strategy Nash Equilibrium
CE	Correlated Equilibrium

Table 2: Strategic form representation of the Heterogeneous Spectrum Sharing game with strategies a_k and a_j.

	a_k	a_j
a_k	$(0,0)$	(u_k, u_j)
a_j	(u_j, u_k)	$(0,0)$

Table 3: Joint prob. distribution over strategies a_1 and a_2.

	a_1	a_2
a_1	$p_{1,1}$	$p_{1,2}$
a_2	$p_{2,1}$	$p_{2,2}$

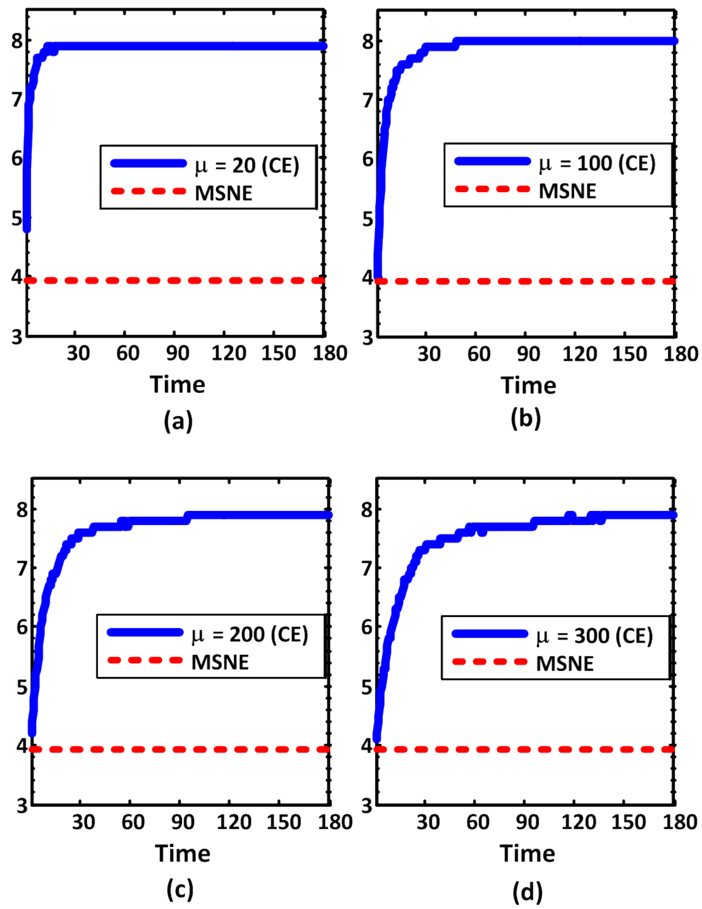

Figure 3: Expected utilities per CRN for $N = 2, K = 2$ and utilities from the two channels are: $u_1 = 9$ and $u_2 = 7$ with varying inertia parameter μ. (a) $\mu = 20$, (b) $\mu = 100$, (c) $\mu = 200$ and (d) $\mu = 300$. Different values of μ achieve the same convergence value of expected utility however as inertia increases, it causes a decrease in convergence rate.

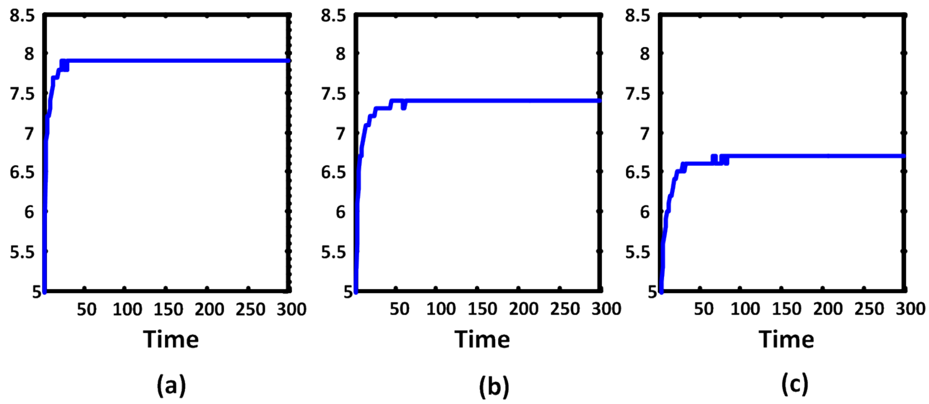

Figure 4: Comparison of CE at different values of the number of networks (n) and channels (k). Y-axes represent expected utility per CRN. For this simulation $n = k$ where (a) $n = k = 2$, (b) $n = k = 3$, (c) $n = k = 4$ such that $u_1 = 9$, $u_2 = 7$ and $u_3 = 6$.

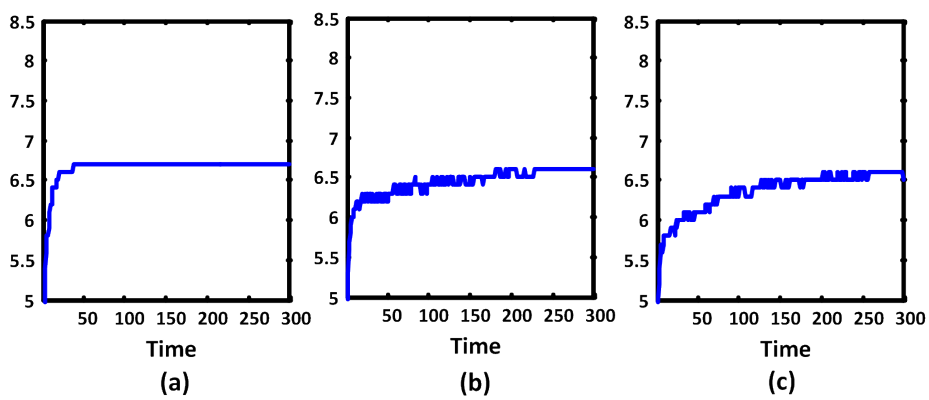

Figure 5: Comparison of CE when $k \geq n$ and number of networks is kept fixed. Y-axes represent expected utility per CRN. (a) $k = n = 4$, (b) $k = 5, n = 4$, (c) $k = 6, n = 4$. CRNs always select the best out of the available pool of channels therefore the convergence value of expected utilities are equal however convergence rate increase as $n \to k$.

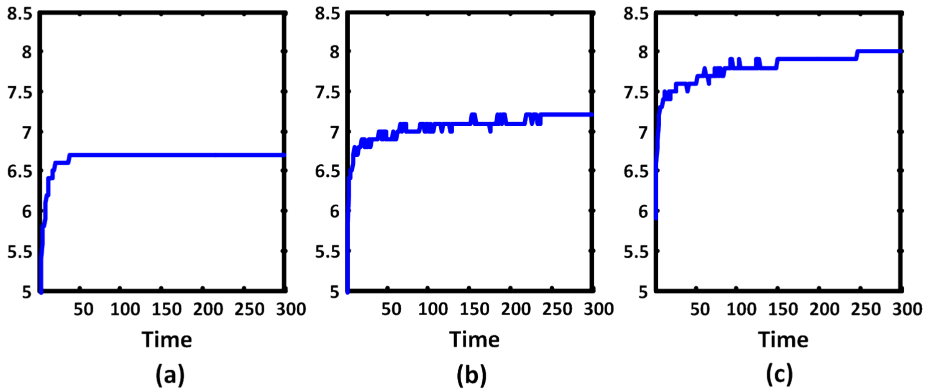

Figure 6: $n \leq k$ and number of networks is kept fixed. Y-axes represent expected utility per CRN. (a) $k = n = 4$, (b) $k = 3, n = 4$, (c) $k = 2, n = 4$. Decrease in number of networks results in an increase in expected utilities. Convergence rate decreases as the number of channels increases.

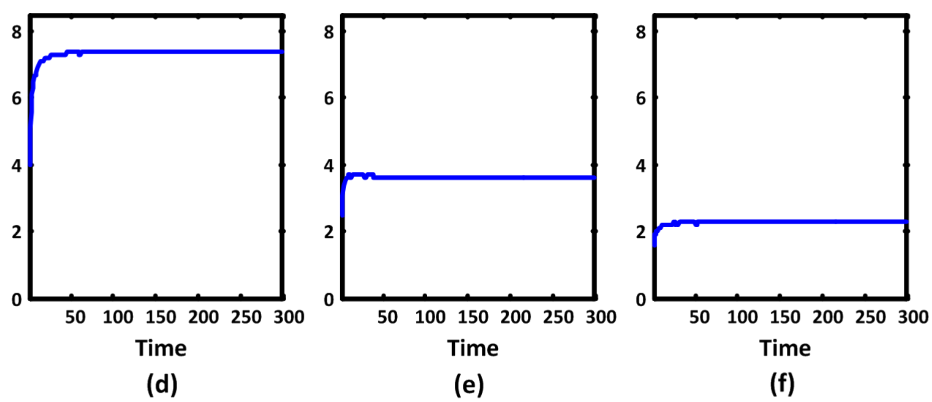

Figure 7: Comparison of CE when $n \geq k$ and number of channels is kept fixed. Y-axes represent expected utility per CRN. (d) $k = n = 2$, (e) $k = 2, n = 3$, (f) $k = 2, n = 4$. Increase in number of networks results in a corresponding decrease in expected utility per CRN however the convergence rate decreases as the number of channels increases.

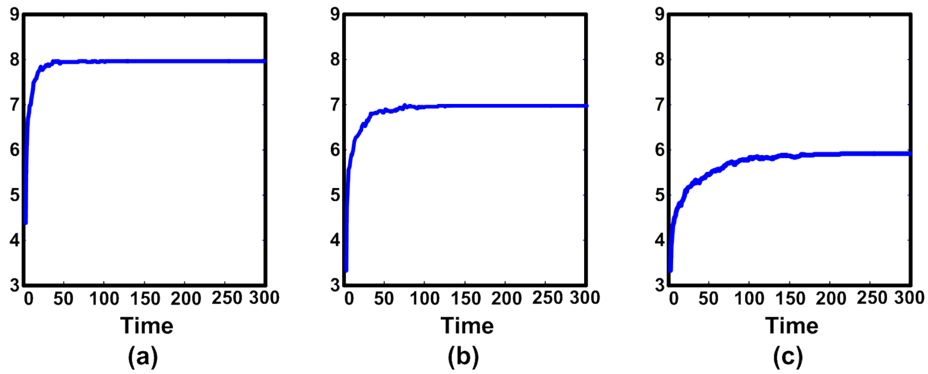

Figure 8: Comparison of CE values when channel payoffs (quality of channels) are varied. Y-axes represent expected utility per CRN with $k = n = 2$, (a) $u_1 = 9, u_2 = 7$, (b) $u_1 = 9, u_2 = 5$ and (c) $u_1 = 9, u_2 = 3$ Decrease in the quality of channel 2 results in a proportional decrease in the expected utility for every CRN. Also, the convergence rate decreases as the variance in channel quality increases.

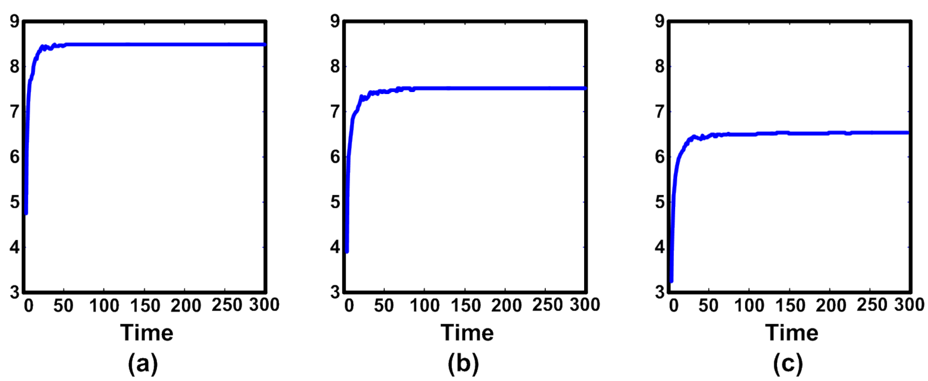

Figure 9: Comparison of CE values when channel payoffs (quality of channels) are varied. Y-axes represent expected utility per CRN with $k = n = 3$, (a) $u_1 = 9, u_2 = 8, u_3 = 7$, (b) $u_1 = 9, u_2 = 8, u_3 = 4$ and (c) $u_1 = 9, u_2 = 8, u_3 = 1$ Decrease in the quality of channel 3 results in a proportional decrease in the expected utility for every CRN. Also, the convergence rate decreases as the variance in channel quality increases.