A Game Theoretic Approach for Energy-Efficient Clustering in Wireless Sensor Networks

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***Abstract*—Selection of clusterheads using energy efficient clustering algorithms in wireless sensor networks (WSNs) is very crucial as it affects the lifetime and performance of the network. As clusterheads and cluster members (i.e., non-clusterheads) expend different amounts energy, it is necessary that all nodes resort to some rational scheme such that the connectivity and proper functioning of the network is not compromised. In this paper, we propose a Cost and Payment-based clustering algorithm (CoPA) for energy efficiency in wireless sensor networks under a game theoretical framework. The analysis is based on a non-cooperative, repeated general-sum game, where each node behaves selfishly in order to maximize its lifespan (payoff). We demonstrate that the correlated equilibrium is a practical solution for clusterhead selection which provides better performance than the Nash Equilibria. Correlated equilibrium provides a balance between the fully cooperative solution and the fully non-cooperative solution in terms of implementation overhead. CoPA produces a balanced distribution of responsibilities and energy consumption between the sensor nodes as well as maximizes the minimum payoff for every node. Results show that CoPA achieves better performance in terms of network lifetime and throughput compared to other popular clustering techniques.**

I. Introduction

Applications of wireless sensor networks (WSNs) can be found in a wide variety of fields such as medical care, transportation, military, home security, and industrial. A typical WSN consists of a large number of sensors that have unique characteristics such as autonomy, limited energy, limited processing capability, and contested radio environment which make their tasks of sensing and communicating difficult [1]. Consequently, energy efficient mechanisms are employed at various layers of the protocol stack that ensure longer longevity for the nodes and the network in general.

Clustering is a grouping technique where a network is partitioned into several clusters– each of which has a clusterhead [2]. The clusterhead is responsible for efficient communication between its cluster members and across other clusters. Typically, a cluster member would communicate with its clusterhead which in turn will communicate with other clusterheads or the base station (sink) of the network. Thus, the identification of clusterheads must be done is such a way so that it prolongs the lifetime of the network and improves the overall scalability of the network.

In this paper, we take a game theoretic approach to devise a clustering algorithm for WSNs. Game theory is a powerful mathematical tool that has been applied to numerous areas of wireless communications for analyzing and predicting the rational and selfish behaviors of various entities– the decisions of which determine the outcome of the game [3]. In our approach, the nodes are the players who play the clustering game. We propose a Cost and Payment-based clustering algorithm (CoPA) where we formalize the profits and losses for each node. CoPA has the provision to alternate the responsibility of a clusterhead among the nodes, thereby balancing energy using a weighted metric that combines the transmission power and energy of each node. We formulate an anti-coordination clustering game for 2 players as well as N players using only local information. We derive the correlated equilibrium (CE) for the clustering game by solving the linear optimization. We use adaptive regret matching (no-regret) algorithm to guarantee convergence of the probability distribution to the CE. Furthermore, we prove and discuss the optimality of CE solution for the clustering game, and compare it to the pure and mixed strategy Nash Equilibria (MSNE) solutions in terms of the efficiency and fairness among the nodes. Finally, we evaluate the performance of our clustering algorithm with two popular clustering techniques, and demonstrate that CoPA has superior performance in terms of network lifetime and system throughput.

II. Related Work

Clustering in wireless sensor networks is an interesting topic especially when it is studied under the game theoretic framework. Various clustering algorithms have been proposed such as the well-known LEACH [4] where the mechanism of selecting a clusterhead is to ensure rotation of the roles between the nodes in a probabilistic manner.

In [5], the authors proposed a an energy-efficient Adaptive Clustering Hierarchy routing algorithm based on game theory. The clusterhead selection is centralized and decided by the base station based on the locations and remaining energy of the nodes. The authors show that the algorithm is suitable for the statically distributed WSNs. However, no theoretical analysis has been provided beside the centralized selection mechanism that could lead to higher energy consumption. In CROSS [2], each sensor behaves selfishly in a non-cooperative manner in order to conserve its energy. The authors provided the pure and mixed strategy NE and the related expected payoffs of the games. The possibility clusterhead absence could occur continuously because of the dependency on selecting the clusterheads based on each node's probability. In [6], the authors proposed a clustering algorithm based on game theory for energy efficiency in wireless sensor network. Furthermore, game theory based energy efficient CH selection approach is proposed in [7] based on the Subgame Perfect NE (SPNE). The clusterheads are selected based on SPNE decision.

In our work, we attempt to provide a new solution from a game theory prescriptive for clustering in WSNs where we study the correlated equilibrium and its properties. The CE achieves strictly better performance compared to the NE and therefore maximizes the network lifetime and throughput.

III. Network Model

We consider a network with N sensor nodes represented by the set $N = \{1, 2, 3, ..n\}$, and divide the entire network into non-overlapping clusters. Each cluster has one clusterhead that receives/transmits data packets from its cluster members and also communicates with the sink in order to deliver those data packets. Furthermore, we consider that the base station is located outside the sensing field. Apart from the communications, the clusterhead has additional responsibilities than the cluster members which include aggregating (i.e., multiplexing and demultiplexing) the data of its members, packet forwarding, and sometimes scheduling. Therefore, on an average, the clusterheads expends more energy. This leads to the situation where each node prefers not to be a clusterhead as long there are others nodes willing to serve as clusterheads. In case all the nodes decide to be cluster members (i.e., no clusterheads), then the data of all cluster members cannot be relayed to the sink which beats the purpose of having a sensor network. Thus, to keep the networking operating in a fair manner [8], the nodes must find it beneficial to rotate roles.

IV. Clustering Game

Let us formally define the game and the cost functions of the nodes. Then we will analyse the equilibria and the no-regret learning for the correlated equilibria.

A. Game Framework

We formulate an anti-coordination $N-$player and $2-$strategy symmetric game. The game is presented as $G = \{N, S, U\}$. The players are represented by N; each player has the same action/strategy space represented by S, and their utility is given by U. The set of strategies available to a sensor node is to decide between being a clusterhead (CH) or a cluster member (CM), and is represented as $S = \{CH, CM\}$. The structure of network is described as a cost and payment model: the nodes gain a specific payoff when they select one of these strategies. Each node behaves selfishly in order to maximize its own payoff (minimize the cost) and stay alive as long as possible. A player may choose to serve as the clusterhead and carry out the additional responsibilities for its members, or refuse to be a clusterhead (e.g., prefer to be a cluster member) in order to maximize its payoff. If more than one player in close physical proximity opt to become a clusterhead, then smaller clusters emerge. As a result, unnecessary control overhead and power consumption would be incurred. However, if none of the nodes opt to be a clusterhead, all the nodes will suffer and all will obtain a payoff of 0 as the nodes will not be able to send their data to the base station. The set of utility functions of the nodes denoted by $U(s_i)$ is given by:

$$U(s_i) = \begin{cases} 0 & \text{when} \quad s_i = CM, \forall i \in N \\ \frac{1}{C_{ch}} & \text{when} \quad s_i = CH \\ \frac{1}{C_{cm}} & \text{when} \quad s_i = CM \end{cases} \tag{1}$$

For the sake of simplicity, let us first provide the possible equilibria in the case of 2 players and their payoffs as presented in Table I. Based on this payoff matrix, the best outcome occurs when one of the nodes selects to be a clusterhead and the other selects to be a cluster member.

TABLE I: Strategic form of 2-player clustering game with strategies CH and CM.

	CH	CM
CH	$\frac{1}{C_{ch}}$, $\frac{1}{C_{ch}}$	$\frac{1}{C_{ch}}$, $\frac{1}{C_{cm}}$
CM	$\frac{1}{C_{cm}}$, $\frac{1}{C_{ch}}$	0 , 0

B. Cost Model

The total cost of being a clusterhead, C_{ch}, consists of two parts: i) the energy spent to transmit packets to the base station and ii) the energy consumed for aggregating the packets received from the cluster members. Thus,

$$C_{ch} = C_{tx(ch,BS)} + C_{rx,aggr} \tag{2}$$

where $C_{tx(ch,BS)}$ is the cost of transmission from the clusterhead to the base station, and $C_{rx,aggr}$ is the cost of receiving and aggregating the packets from the cluster members. We define $C_{tx(ch,BS)}$ as:

$$C_{tx(ch,BS)} = d^2_{ch,BS} \cdot e_{amp} + e_{elec} \tag{3}$$

where $d_{ch,BS}$ is the distance between the clusterhead and the base station, e_{amp} the transmit amplifier dissipation in order to achieve the required signal level, and e_{elec} is the transmission circuitry dissipation.

As for the cost for receiving and aggregating, it proportional to the cluster size (i.e., $\bar{k}$ average number of neighbors), i.e.,

$$C_{rx,aggr} \propto \bar{k} \tag{4}$$

It is to be noted that the cluster members will be at varying distances from the clusterhead and therefore the clusterhead uses different power levels to transmit to its members. (We assume that there is some power control algorithm is place– the specifics of which is beyond the scope of this paper.) Thus,

$$C_{rx,aggr} = \sum_{i=1}^{\bar{k}} d_i^2 \cdot e_{elec} + \bar{k} \cdot e_{aggr} + e_{lis} \tag{5}$$

where d_i is the distance of the ith cluster member from its clusterhead and e_{aggr} is the cost of aggregation for one cluster member. e_{lis} is the cost of listening to the wireless medium even though no packets are being to transmitted. The cost of being a cluster member is the cost of transmission from node i to its clusterhead ch_i considering the distance (d_{i,ch_i}) is calculated by:

$$C_{cm} = C_{tx(i,ch_i)} = e_{amp}.d^2_{i,ch_i} + e_{elec} \tag{6}$$

According to above mentioned energy model and assuming the base station is located outside the sensing region, the cost of being a clusterhead is expected to be larger than the cost of being a cluster member, i.e.,

$$C_{ch} > C_{cm} \tag{7}$$

C. Analysis and Equilibrium

1) Pure and Mixed Nash Equilibrium:

For the clustering game, we derive the solution concepts in the form of Pure and Mixed Nash Equilibrium for 2-players and N-players.

Lemma 1: Strategy pairs (CH, CM) and (CM, CH) are pure strategy NE for 2-player clustering game.

Proof: In Table I, assume the row and the column are the two players from the cluster. These players select strategy pairs (CH, CM) and (CM, CH). The payoffs of the selection are $(\frac{1}{C_{ch}}, \frac{1}{C_{cm}})$ and $(\frac{1}{C_{cm}}, \frac{1}{C_{ch}})$, respectively. Let us say that the players select strategy pairs (CH, CH) and (CM, CM)

instead. Thus, the payoffs for those strategy pairs will be $(\frac{1}{C_{ch}}, \frac{1}{C_{ch}})$ and zero, respectively. This means that the player who is playing strategy CH does not have an incentive to change the strategy to CM because of receiving less payoffs (i.e., zero). Furthermore, the player who is playing strategy CM does not have an incentive to change the strategy to CH because of receiving less payoffs too (i.e., $\frac{1}{C_{ch}}$). Thus, the strategy pairs (CH, CM) and (CM, CH) are a pure NE for this game according to the definition [9]. ∎

Proposition 1: For the anti-coordination clustering game for N players, there are N pure NE where the strategy of a single player is to select CH and all the rest of the nodes to select CM.

The mixed strategy Nash Equilibrium of the clustering game is a probability distribution $\hat{p}$ over the pure NE where each player will have equal expected payoff. Each node will take a random selection conformity with the probability distribution. Let α be the probability of playing CH and $\beta = 1 - \alpha$ be the probability of playing CM. In order to compute these probabilities, we calculate the expected utility function of playing CH as:

$$U_{CH} = \frac{1}{C_{ch}} \tag{8}$$

The expected utility of playing CM is obtained by:

$$U_{CM} = \frac{1}{C_{cm}} \cdot [1 - (1 - \alpha)^{N-1}] \tag{9}$$

According to the definition of mixed NE [10], the expected utilities of playing strategies CH and CM are equal and no player has incentive to change her strategy. Thus,

$$U_{CH} = U_{CM} \tag{10}$$

Substituting (8) and (9) in (10) and solving the expression in order to calculate the probability α that corresponds to the equilibrium, we get:

$$\alpha = 1 - (\frac{C_{ch} - C_{cm}}{C_{ch}})^{\frac{1}{N-1}} \tag{11}$$

The distribution of the mixed strategy NE for the clustering game is $\hat{p} = \{\alpha, \beta\}$ which means that the players will mix their choice for selecting clusterhead strategy and cluster member without incurious about the outcome. However, MSNE is not efficient enough where we could end up with (CH, CH) or (CM, CM) strategies, which is not desirable for the system and could lead to performance degradation of the network.

2) Correlated Equilibrium (CE):

We propose a new solution concept, Correlated Equilibrium, for the clustering game that maximizes the outcome and prevents undesirable action. The correlated equilibrium concept is more general than NE and was first proposed by Nobel Laureate Robbert J. Aummann [11].

Thus far, the players' strategies are independent where each player chooses her mixed strategy independently without any communication with each other. According to MSNE solution, the player will gain equal payoffs. However, if the player can avoid ending up with the same strategies by following agreement/external signal for the coordination of actions between the nodes, the outcome will maximize, and efficiency of the system will be higher. The strategy profile is selection according to joint distribution. This resulting distribution strategy profile called Correlate Equilibrium, where it is best interest for the player to follow the external signal and conform with the recommended strategy. Thereby, the players have no incentive to deviate to gain higher payoff.

The essence of a correlated equilibrium [9] is that when all players follow the external recommendation signal, no player has a unilateral incentive to deviate from the trusted authority's recommendation to achieve higher payoff. Moreover, that signal could be generated by an arbitrator which is seen as a virtual entity and does not depend on the system. The correlated equilibrium is defined as:

Definition of correlated equilibrium [9]: A probability distribution π is a correlated equilibrium of the game G if and only if, for all $i \in N$, $s_i \in S_i$ and $s_{-i} \in S_{-i}$:

$$\sum_{s_{-i} \in S_{-i}} \pi(s_i, s_{-i})[u_i(s'_i, s_{-i}) - u_i(s_i, s_{-i})] \leq 0 \tag{12}$$

where $\pi(s_i, s_{-i})$ denotes the joint probability distribution of players. The action for user i and its opponents are s_i and s_{-i} The inequality (12) implies that the expected payoff of player i playing the recommendation strategy s_i at the CE is greater than or equal to the expected payoff that could be received for choosing any other strategy s'_i. In other words, choosing action s'_i instead of s_i cannot obtain a higher expected payoff for user i.

3) Linear Programming Solution :

For the proposed game, we investigate a linear optimization method to calculate the optimal CE [9]- [11]. We drive the CE linear system for 2-player game as shown in Table I, then we implement the same mechanism for N players. A correlated strategy pair in the game is given by the CE joint probability distribution which presented as 4-dimensional vector $\pi = (p_1, p_2, p_3, p_4)$, where $p_1 + p_2 + p_3 + p_4 = 1$. A correlated strategy pair means that the strategy pair (CH, CH) is played with probability p_1, strategy pair (CH, CM) is played with probability p_2, strategy pair (CM, CH) is played with probability p_3, and strategy pair (CM, CM) is played with probability p_4.

In order to find the egalitarian equilibrium for the game, we formulate the game as linear programming and define the objective function f to find the optimal strategy CE as:

$$f = \max_{p} \sum_{i \in N} \mathbb{E}_p(u_i) \tag{13}$$

$$\text{such that} \begin{cases} \forall s_i, s'_i \in S_i, \text{and}, i \in N, \\ p(s_i, s_{-i})[u_i(s'_i, s_{-i}) - u_i(s_i, s_{-i})] \leq 0 \end{cases}$$

where $\mathbb{E}_p(.)$ is the expectation over p. Then, the constrains for CE for 2-player game are:

$$u_1(CH, CH)p_1 + u_1(CH, CM)p_2 \geq u_1(CH, CH)p_1 + u_1(CM, CM)p_2 \tag{14}$$

$$u_1(CM, CH)p_3 + u_1(CM, CM)p_4 \geq u_1(CH, CH)p_3 + u_1(CH, CM)p_4 \tag{15}$$

$$u_2(CH, CH)p_1 + u_2(CM, CH)p_3 \geq u_2(CH, CM)p_1 + u_2(CM, CM)p_3 \tag{16}$$

$$u_2(CH, CM)p_2 + u_2(CM, CH)p_4 \geq u_2(CH, CH)p_1 + u_2(CM, CH)p_4 \tag{17}$$

By solving the above inequalities, the obvious solution for the CE probability distribution is: $p_1 = p_4 = 0$ and $p_2 = q, p_3 = 1 - q$ which maximizes the sum of the

TABLE II: An example of payoffs matrix for 2-player

	CH	CM
CH	$\frac{1}{6}$, $\frac{1}{6}$	$\frac{1}{6}$, $\frac{1}{2}$
CM	$\frac{1}{2}$, $\frac{1}{6}$	0 , 0

expected payoffs for all players. Thus, the CE joint probability distribution $\pi = (0, q, 1-q, 0)$. Thereby, we eliminated the possibility of selecting the same strategy for the players.

For N-player and 2-strategies clustering game, we can derive the linear system and CE constrains according to (13) in the same manner for obtaining polynomial time algorithms for optimizing over CE. The inequalities constraints grow exponentially with the number of players [12]. This result proves that following the external signal is self-enforcing, since cooperation arises naturally from the rules of the game. In addition, it must be considered that the external signal is not binding and players can ignore it. Thus, we guarantee the convergence of equilibrium to CE by utilizing the no-regret learning algorithm discussed in section IV-E.

D. *Fairness and Efficiency (Pareto Optimality)*

In this subsection, we will discuss the fairness and efficiency of all the proposed solution game (i.e., Pure and mixed strategy NE compared with CE), as well as evaluate the proposed CE solution by applying the Pareto optimality concept. Pareto Optimality is the objective measurement of efficiency in game theory.

The two pure strategy NE in th clustering game (i.e., (CH, CM) and (CM, CH)) are unfair where one node always gets higher payoff than other. However, the MSNE for the game achieves the fairness where the expected utility of the players are equal. For sake of clarity, let us assume the example of payoffs matrix for 2-players as shown in Table II. The MSNE for the clustering game is the distribution $(\alpha = \frac{1}{3}, \beta = \frac{2}{3})$ over the set of pure strategies. The expected utility for both players will be equal when they mix their strategies according to MSNE. As per equations (8)-(11), the expected utility is 0.16. Additionally, the chance of none of the players being a clusterhead $(\frac{2}{3} \times \frac{2}{3} = 44.4\%)$, and the chance of ending up with more than one clusterhead at the same time is $(\frac{1}{3} \times \frac{1}{3} = 11.1\%)$. This means that there is always a high chance of an undesirable action occurring with MSNE (i.e., 55%) either for lost communication with base station in the case of absence the clusterhead, or energy wastage in case of more than one clusterhead in the cluster. Accordingly, the MSNE is an inefficient equilibrium to the game. In the same manner, the joint probability distribution of CE for the game shown in Table II is $(\pi = \{0, \frac{1}{2}, \frac{1}{2}, 0\})$ which is calculated by the inequalities (13-17). The expected utility for the players is $\frac{1}{2} \times (\frac{1}{6} + \frac{1}{2}) = 0.33$, which is greater than the expected utility of MSNE as well as the payoffs of always be a clusterhead. Furthermore, another way to prove the efficiency of the CE is to analyze the Pareto optimality of the solution. The main idea of Pareto optimality is to maximize the outcome of the game where no player can be better off without making some player worse off. In other words, an outcome of the game is Pareto efficient if there is no other outcome where some player's utility can be increased without making some other player's utility worse [13]. Figure 1 is the convex hull graphical presentation of the game considered in Table II. The 4 points in the figure represent the possible payoffs. The maximum payoffs attainable by a node must occur at one of the vertices of the convex hull (i.e., (CH, CM) and (CM, CH)), which are the pure Nash equilibria. It can be noticed that the set of Pareto optimal solution is the line between these two payoff vectors, whereas a mix between these two vectors is the proposed CE solution for the game.

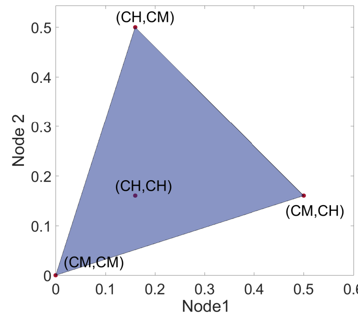

Fig. 1: Geometrical representation of the set of attainable payoffs under CE for Table II

Therefore, the CE is Pareto optimal (i.e., efficient) solution, where it maximizes the expected utility besides achieving fairness Moreover, it must be noticed that the CE is less expensive than NE computationally, where computing CE only requires solving a linear program. In contrast, NE requires finding its fixed point completely to solve it.

E. *No-Regret Learning Algorithm for CE N-player game*

We provide how the strategies of the players reach an equilibrium without needing the trust arbitrators, where the recommended signal is not binding and the players are free to ignore it. In order for the convergence to occur to the set of correlated equilibria in the long run, we use the learning process called regret matching (no-regret) algorithm [14]. The goals of the algorithm is to minimize the regret of each player and reach 0 as time $t \to \infty$. Adjustment of the probability distribution is guided by the average difference (i.e., regret measures) based on the history of the actions that have been played by all players from past periods.

In particular, assume that the game is played repeatedly through time $t \in \{1, 2, 3, 4, ...T\}$, and player i selects the distribution $(p_t)_i$ over action S. Each player in each period decides either to continue playing the same probability distribution $(p_t)_i$ for next time $(t+1)$ or switch to other probabilities $(p'_t)_i$ that are proportional to the difference "regrets" relative to the current probability. Precisely, for any two distinct actions $s, s' \in S^i$ of player i selecting according to a probability distribution $p^i_t(s)$ and $p^i_t(s')$, respectively, the regret of the player i at time T for not playing s'_i is calculated as:

$$R^i_t(s_i, s'_i) = max\{D^i_t(s_i, s'_i), 0\} \tag{18}$$

where the average difference is given by:

$$D^i_t(s_i, s'_i) = \frac{1}{t} \sum_{\tau=1}^{t} [u_i(s'_i, s^{-i}_\tau) - u^i(s_\tau)] \tag{19}$$

For next period $(t+1)$, the probability $p^i_{t+1}(s_i)$ and $p^i_{t+1}(s'_i)$ for player i to take action s_i and s'_i, respectively, are computed as:

TABLE III: Regret-matching (no-regret) learning algorithm

Initialization: Set the probability for taking action $s_i, \forall s \in S_i$ for the node i arbitrarily, $p^i_{t=1}(s_i)$

for $t = 1, 2, 3, 4...$
for each node i
Calculate payoff u^i_t for playing with probability $p^i_t(s_i)$
Find the regret $R^i_t(s_i, s'_i)$ of the player i for not playing s'_i up to time t (equations (18-19)).
Find the probability distribution action for $t+1$ (equation (20)) as:
1. Update $p^i_{t+1}(s_i)$ to take action s_i
2. Calculate $p^i_{t+1}(s'_i)$ to take action s'_i
end
end

$$\begin{cases} p^i_{t+1}(s'_i) = \frac{1}{\mu} R^i_t(s_i, s'_i), \\ p^i_{t+1}(s_i) = 1 - p^i_{t+1}(s'_i). \end{cases} \tag{20}$$

where the probability $p^i_{t+1}(s_i)$ is a linear function of regret, and μ is an independent parameter of time and history, and is sufficiently large. Choice of $\mu > 2M^i$ guarantees that the probability of playing the same strategy as in the last period is positive, where M^i is an upper bound on $|u^i(.)|$. In each period, the player selects an action and observes the loss/gain to adjust the probability of choosing an alternative action for higher payoff until the strategies converge to CE. Table III shows the summary of no-regret learning algorithm.

V. Strategy Space Reduction for CoPA

Though all the nodes in the network should contribute to the network by serving as clusterhead from time to time, there would always be some nodes that are less suitable to take on the added responsibility. At any point of time, there would be better suited nodes and 'weaker' nodes. The weaker nodes might have less energy remaining, lower transmission capabilities, or lower computing power. Therefore, instead of having *all* nodes participate in the game and exploring the entire strategy space for finding the equilibrium solution, we argue that certain weaker nodes can safely be excluded for clusterhead consideration.

In order to select the group of nodes that will contribute into the game at any time instance, we consider two system parameters– transmission energy consumption and residual energy, and combine them using a weighed average. If ω_n represents the weighted average of node n, then

$$\omega_n = w_1 D_n + w_2 E_n \tag{21}$$

where D_n is the summation of the distances of all neighbours of node n (i.e., $D_n = \sum_{n \in N} \{dist(n, n')\}$), and E_n denotes how much energy the node consumed till current time. w_1 and w_2 are the weighting factors.

Based on ω_n, it is relatively easy to categorize a node as 'suitable' or 'unsuitable' just by comparing ω_n to some $threshold$ value. As for determining the $threshold$, a simple way would be to use some local cluster parameters, like the mean of the weighted average of all the nodes. Additionally, the $threshold$ is updated periodically and sent to all cluster

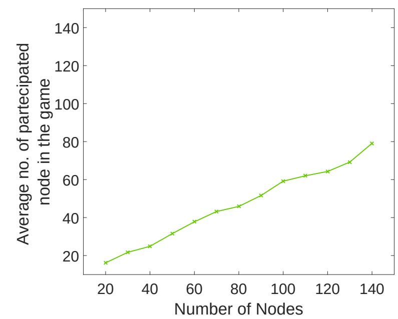

Fig. 2: Number of nodes that participate in our proposed clustering game (CoPA).

members by the same arbitrator (i.e., virtual entity) responsible for generating the external signal for CE solution.

The suitable nodes participate in the repeated clustering game by playing the game in rounds. After each round, all nodes update ω_n and compare with the new $threshold$ for the next round. This exclusion policy has two main features: i) the weighted metric is generic enough and can accommodate any number of node parameters, ii) prohibits unsuitable nodes to participate in the game, thereby reducing the strategy space and speeding up the equilibrium convergence.

VI. Performance Evaluation

In order to test the veracity of of the correlated equilibrium in determining the clusterhead set in a wireless sensor network, we resort to simulation experiments.

A. Simulation Setup

We simulate a system of N sensor nodes using MATLAB. In order to measure the performance of our clustering algorithm, we compare it with two clustering techniques: probability-based [15] and CROSS [2]. The probability of being a clusterhead is fixed in the probability-based, and is set to 0.05 as in [15]. The probability of being a clusterhead in CROSS is defined as $p = 1 - \omega^{\frac{1}{N-1}}, 0 < \omega < 1$; where the value of ω is set as per [2]. For CoPA, the probability of being a clusterhead or a cluster member depends on the CE probability distribution for the clustering game as presented in Section IV. We assume that the base station is located outside the sensing field. The sensor nodes form a connected network i.e., we get a single component graph. The rest of the simulation parameters are presented in Table IV. Furthermore, we identify three metrics that reveal the performance of any clustering technique: network lifetime, average residual energy, and amount of data sent to the sink (throughput).

TABLE IV: Simulation Parameters

Parameters	Value
Initial energy	0.5 J
Transmit and receive energy	50 nJ
Transmit to the base station	100 nJ
Data aggregation energy	5 nJ

B. Simulation Results

In order to show the relative performance of exclusion policy of CoPA, Fig. 2 presents the number of nodes that contributes to the game for a various number of sensor nodes

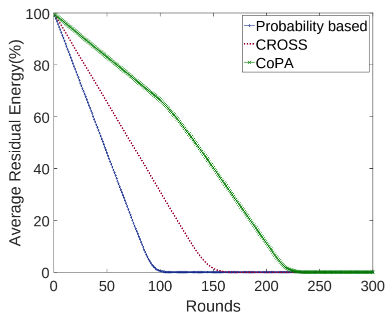

Fig. 3: Average residual energy

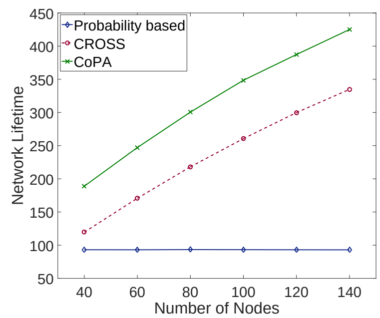

Fig. 4: Network Lifetime

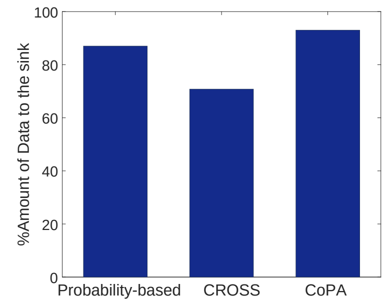

Fig. 5: Amount of data sent to the BS

(i.e., $N = 20, ..., 140$). We notice that the average number of participated nodes is less than the total number of sensor nodes (i.e., $55\% - 65\%$). This is because of the exclusion policy (Section V), which decides which sensor nodes will be excluded from being considered in the game. Therefore, the strategy space will significantly reduce and the equilibrium convergence will speed up.

Fig. 3 shows the average residual energy of the sensor nodes for the three clustering methods. The number of nodes considered in this experiment was 50. For the probability-based and CROSS clustering, the average residual energy for the nodes drops to almost 0 in 100 and 150 rounds, respectively. CoPA on the other hand has a steadier energy degradation. Fig. 4 exhibits the network lifetime for various number of sensor nodes (i.e., $N = \{40, ...140\}$) for the probability-based, CROSS, and CoPA. We define network lifetime as 'the lifespan of the node that first among all the others deplete its energy' [2]. We consider a node's energy is exhausted when 99% of the sensors' initial energy has been consumed. CoPA achieves longer lifetime than the other two for any number of nodes.

We also measure the throughput of the system according to the amount of data sent to the sink (base station), where the only way to reach the base station is through the clusterheads. In the absence of any clusterhead, the data cannot be relayed to the base station. In Fig. 5, we present the amount of data that was sent to the base station. It is interesting to observe that CoPA has the highest value, which is 5% and 20% more than the probability-based and CROSS, respectively. Consequently, CoPA ensures of determination of clusterheads in each round and guarantees a pathway for the sensed data to be sent to the base station. As a final comment, the absence of clusterhead could occur continuously in the probability-based and CROSS because of their dependence on the node's probability for playing as a clusterhead, whereas CoPA guarantees of the existence of clusterheads in every round till the network dies.

VII. Conclusions

In this work, we proposed a cost and payment clustering techniques for wireless sensor networks. CoPA determines the cost of being a clusterhead or a cluster member and provides the probability distribution for the correlated equilibrium. In addition, we proved that the correlated equilibrium achieves better performance than the pure and mixed strategy Nash equlibria in term of efficiency and fairness. We also proposed a simple way to determine a node's eligibility to participate in the clustering game based on a flexible weighted function. The unsuitable nodes are prohibited; thereby reducing the strategy space and speeding up convergence to the equilibrium. Simulation results show that CoPA outperforms probability-based and CROSS clustering in terms of residual energy, network lifetime, and throughput.

References

[1] J. A. Stankovic, A. D. Wood, and T. He, "Realistic applications for wireless sensor networks," in *Theoretical Aspects of Distributed Computing in Sensor Networks*, pp. 835–863, Springer, 2011.

[2] G. Koltsidas and F.-N. Pavlidou, "A game theoretical approach to clustering of ad-hoc and sensor networks," *Telecommunication Systems*, vol. 47, no. 1-2, pp. 81–93, 2011.

[3] W. Wang, M. Chatterjee, and K. Kwiat, "Coexistence with malicious nodes: A game theoretic approach," in *Game Theory for Networks. GameNets. International Conference on*, pp. 277–286, IEEE, 2009.

[4] W. R. Heinzelman, A. Chandrakasan, and H. Balakrishnan, "Energy-efficient communication protocol for wireless microsensor networks," in *System sciences.*, pp. 10–pp, IEEE, 2000.

[5] Z. Zeng-Wei, W. Zhao-Hui, and L. Huai-Zhong, "Clustering routing algorithm using game-theoretic techniques for wsns," in *Circuits and Systems. ISCAS.*, vol. 4, pp. IV–904–7 Vol.4, May 2004.

[6] M. Esmaeeli and S. A. H. Ghahroudi, "Improving energy efficiency using a new game theory algorithm for wireless sensor networks," *International Journal of Computer Applications*, vol. 136, no. 12, 2016.

[7] M. Mishra, C. R. Panigrahi, J. L. Sarkar, and B. Pati, "Gecsa: A game theory based energy efficient cluster-head selection approach in wireless sensor networks," in *International Conference on Man and Machine Interfacing (MAMI)*, pp. 1–5, Dec 2015.

[8] S. Brahma, M. Chatterjee, and K. Kwiat, "Congestion control and fairness in wireless sensor networks," in *Pervasive Computing and Communications Workshops (PERCOM Workshops), 8th IEEE International Conference on*, pp. 413–418, IEEE, 2010.

[9] Z. Han, *Game Theory in Wireless and Communication Networks: Theory, Models, and Applications*. Game Theory in Wireless and Communication Networks: Theory, Models, and Applications, Cambridge University Press, 2012.

[10] D. Fudenberg and J. Tirole, "Game theory." *Cambridge, Massachusetts*, vol. 393, 1991.

[11] R. J. Aumann *et al.*, "Subjectivity and correlation in randomized strategies," *Journal of mathematical Economics*, vol. 1, no. 1, pp. 67–96, 1974.

[12] C. H. Papadimitriou and T. Roughgarden, "Computing correlated equilibria in multi-player games," *Journal of the ACM (JACM)*, vol. 55, no. 3, p. 14, 2008.

[13] P. M. Pardalos, A. Migdalas, and L. Pitsoulis, *Pareto optimality, game theory and equilibria*, vol. 17. Springer Science & Business Media, 2008.

[14] S. Hart and A. Mas-Colell, "A simple adaptive procedure leading to correlated equilibrium," *Econometrica*, vol. 68, no. 5, pp. 1127–1150, 2000.

[15] W. R. Heinzelman, A. Chandrakasan, and H. Balakrishnan, "Energy-efficient communication protocol for wireless microsensor networks," in *System Sciences.*, p. 10 pp. vol.2, Jan 2000.