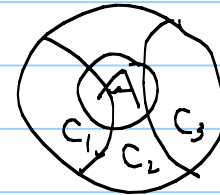


$$\lambda_{n-1}P_{n-1} = \mu_n P_n$$

$$\begin{aligned}
 P_{ij}^{n+m} &= P(X_{n+m} = j \mid X_0 = i), \\
 &= \sum_{k=0}^{\infty} P(\underbrace{X_{n+m} = j, X_n = k}_{A \mid B} \mid X_0 = i), \\
 &= \sum_{k=0}^{\infty} P(\underbrace{X_{n+m} = j \mid X_n = k, X_0 = i}_A) P(\underbrace{X_n = k \mid X_0 = i}_B),
 \end{aligned}$$

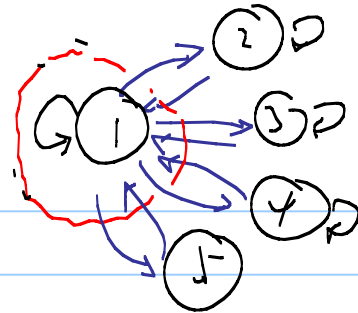


$$P(A) = P(A \cap C_1) + P(A \cap C_2) + P(A \cap C_3)$$

$$P(A|B) = P(A \cap B) \cdot P(B)$$

$$\pi = \pi P,$$

$$\pi \mathbf{1} = 1$$



$$\pi_i \sum_{j \neq i} \lambda_{ij} = \sum_{j \neq i} \pi_j \lambda_{ji}$$

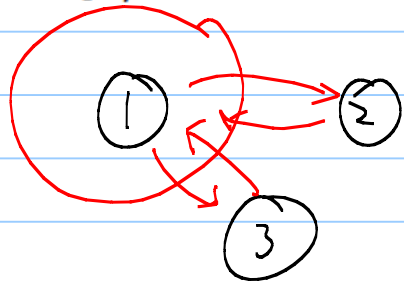
$$\underbrace{\pi_1 \cdot (1 - p_{11})}_{\text{jump out}} = \underbrace{\pi_2 \cdot p_{21} + \pi_3 \cdot p_{31} + \pi_4 \cdot p_{41} + \dots}_{\text{jump in}}$$

$$\begin{bmatrix} \pi_1 \\ \pi_2 \end{bmatrix} = \begin{bmatrix} \pi_1 p_{11} + \pi_2 p_{21} + \pi_3 p_{31} + \dots \\ \pi_1 p_{12} + \pi_2 p_{22} + \pi_3 p_{32} + \dots \end{bmatrix}$$

↓
 $P \pi$

↓
column vector

$$\pi_i \sum_{j \neq i} \lambda_{ij} = \sum_{j \neq i} \pi_j \lambda_{ji}$$



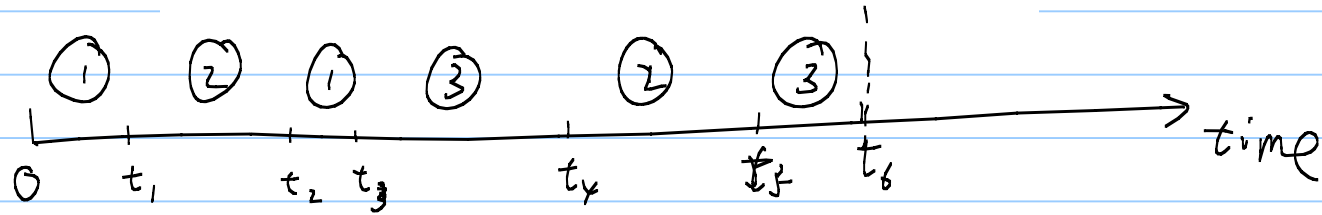
$$\Rightarrow \pi_1 (\lambda_{12} + \lambda_{13}) = \pi_2 \lambda_{21} + \pi_3 \lambda_{31}$$

$$- (\lambda_{12} + \lambda_{13}) \pi_1 + \pi_2 \lambda_{21} + \pi_3 \lambda_{31} = 0$$

$$(\pi_1 \quad \pi_2 \quad \pi_3) \begin{bmatrix} -(\lambda_{12} + \lambda_{13}) \\ \lambda_{21} \\ \lambda_{31} \end{bmatrix} = 0$$

$$Q = \begin{pmatrix} -(\lambda_{12} + \lambda_{13}) & \lambda_{12} & \lambda_{13} \\ \lambda_{21} & -(\lambda_{21} + \lambda_{23}) & \lambda_{23} \\ \lambda_{31} & \lambda_{32} & -(\lambda_{31} + \lambda_{32}) \end{pmatrix} \Rightarrow \pi Q = 0$$

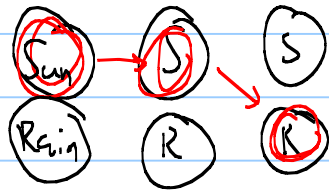
$$\pi_j = \frac{\pi_j' E[Y^{(j)}]}{\sum_{i \in S} \pi_i' E[Y^{(i)}]}, \quad j \in S$$



$$\pi_1' = \frac{2}{6} = \frac{1}{3}$$

$$\pi_2' = \frac{2}{6}$$

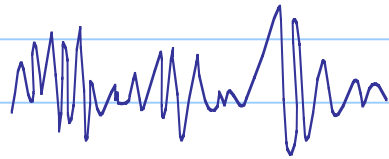
$$\pi_1 = \frac{(t_1 - t_0) + (t_3 - t_2)}{t_6 - t_0}$$



observed \rightarrow F S S F F F S S S

"pop"

← hidden state



← observed