$$\pi_{n-1}\lambda = \pi_n \mu \quad \Rightarrow \pi_n = \rho^n \pi_0$$

Note Title 9/29/2011

$\pi_0 + \rho\pi_0 + \rho^2\pi_0 + \cdots = 1$

$\pi_0 = \dfrac{1}{(1+\rho+\rho^2+\rho^3 \ldots\ldots)} = 1-\rho$

$\dfrac{1}{1-\rho}$

$$E[N] = \sum_{k=1}^{\infty} k\pi_k = \pi_0 \underbrace{\sum_{k=1}^{\infty} k\rho^k}_{=S} = \frac{\rho}{1-\rho}$$

$S = \rho + 2\rho^2 + 3\rho^3 + 4\rho^4 + \cdots\cdots$

$\rho S = \rho^2 + 2\rho^3 + \cancel{3}\rho^4 + 5\rho^4 + \cdots\cdots$

$(1-\rho)S = \rho + \rho^2 + \rho^3 + \rho^4 + \cdots\cdots = \dfrac{\rho}{1-\rho}$

$S = \dfrac{\rho}{(1-\rho)^2}$

$$E[T] = \frac{1}{\mu} + E[W] = \frac{1}{\mu-\lambda}$$

$\rho = \frac{\lambda}{\mu}$

$E[W] = E[N] \cdot E[X] = \frac{\rho}{1-\rho} \cdot \frac{1}{\mu}$

$= \frac{1}{\mu} + \frac{1}{\mu} \cdot \frac{\rho}{1-\rho} = \frac{1}{\mu}\left(1 + \frac{\rho}{1-\rho}\right) = \frac{1}{\mu} \cdot \frac{1}{1-\rho} = \frac{1}{\cancel{\lambda}} \cdot \frac{\cancel{\mu}}{\mu-\lambda}$

$= \frac{1}{\mu-\lambda}$

$\lambda = 80 \quad \mu = 100, \quad \rho = \frac{\lambda}{\mu} = 0.8$

Q1: $E[N] = \frac{\rho}{1-\rho} = \frac{0.8}{0.2} = 4$

Q2: $E[W] = E[N] \cdot \frac{1}{100} = 0.04$ sec

Q3: $E[T] = \frac{1}{\mu-\lambda} = \frac{1}{20}$ sec $= 50$ ms

$\pi_i \rightarrow P(N=i)$

Q4: π_0? $\quad \pi_0 = 1-\rho = 0.2$

Q5: $P(N>5) = \pi_6 + \pi_7 + \pi_8 + \dots = 1 - (\pi_0 + \pi_1 + \pi_2 + \pi_3 + \pi_4 + \pi_5)$

$\pi_i = \rho^i \cdot \pi_0$

Q6 μ? $\Rightarrow$ $E[T] = 20ms$

$$= \frac{1}{\mu - \lambda}$$

$$\frac{1}{\mu - \lambda} = \frac{1}{50} \Rightarrow \mu = 130$$

λ: 80

$\Rightarrow$ Bandwidth 130kbps

$$\boxed{\sum_{i=0}^{\infty} \frac{\alpha^i}{i!} = e^{\alpha}}$$

X_1, X_2, X_3

$Y = \min(X_1, X_2, X_3)$