Computer Science Foundation Exam

August 14, 2009

Section II A

DISCRETE STRUCTURES SOLUTIONS

NO books, notes, or calculators may be used,
and you must work entirely on your own.

Name: ______________________________

PID: ______________________________

In this section of the exam, there are three (3) problems.
You must do ALL of them.
They count for 40% of the Discrete Structures exam grade.
Show the steps of your work carefully.

Problems will be graded based on the completeness of the solution steps and not graded based on the answer alone.

Credit cannot be given when your results are unreadable.

Question #	Max Pts	Category	Passing	Score
1	15	PRF (Induction)	10	
2	15	PRF (Sets)	10	
3	10	PRF (Logic)	6	
ALL	40	---	26	

1) (15 pts) PRF **(**Induction)

Using proof by induction on n, prove that $8 \mid (3^{2n+1} + 5^{2n+1})$ for all non-negative integers n.

Base case: n=0. $3^{2(0)+1} + 5^{2(0)+1} = 3 + 5 = 8$. Since $8 \mid 8$, the base case holds. (2 pts)

Inductive hypothesis: Assume for an arbitrary non-negative integer n = k that $8 \mid (3^{2k+1} + 5^{2k+1})$, namely that there exists some integer c such that $8c = 3^{2k+1} + 5^{2k+1}$. (2 pts)

Inductive step: Prove that for n = k+1 that $8 \mid (3^{2(k+1)+1} + 5^{2(k+1)+1})$. (Namely, prove that there exists some integer d such that $8d = 3^{2(k+1)+1} + 5^{2(k+1)+1}$.) (2 pts)

$3^{2(k+1)+1} + 5^{2(k+1)+1})$	=	**$3^{2k+3} + 5^{2k+3}$**	**(1 pt)**
	=	**$3^2(3^{2k+1}) + 5^2(5^{2k+1})$**	**(2 pts)**
	=	**$9(3^{2k+1}) + 25(5^{2k+1})$**	
	=	**$9(3^{2k+1}) + 9(5^{2k+1}) + 16(5^{2k+1})$**	**(2 pts)**
	=	**$9(3^{2k+1} + 5^{2k+1}) + 16(5^{2k+1})$**	**(1 pt)**
	=	**$9(8c) + 16(5^{2k+1})$, using the inductive hypothesis**	**(2 pts)**
	=	**$8(9c + 2(5^{2k+1}))$, which completes the proof, since both c and k are integers.**	**(1 pt)**

2) (15 pts) PRF **(**Sets)

Use set laws to prove that the two following sets are equivalent.

(1) $A \cup B$ (2) $(A \cap B) \cup (A \cap \bar{B}) \cup (\bar{A} \cap B)$

$(A \cap B) \cup (A \cap \bar{B}) \cup (\bar{A} \cap B) = \big(A \cap (B \cup \bar{B})\big) \cup (\bar{A} \cap B)$, Distributive Law **(3 pts)**
$=(A \cap U) \cup (\bar{A} \cap B)$, Inverse Law **(2 pts)**
$=A \cup (\bar{A} \cap B)$, Identity Law **(2 pts)**
$=(A \cup \bar{A}) \cap (A \cup B)$, Distributive Law **(3 pts)**
$=U \cap (A \cup B)$, Inverse Law **(2 pts)**
$= A \cup B$, Identity Law **(3 pts)**

3) (10 pts) (PRF) Logic

Use the Laws of Logic and Rules of Inference to justify the following argument:

$p \lor q$
$p \rightarrow r$
$q \rightarrow s$
$(r \lor s) \rightarrow t$
$t \rightarrow (u \land v)$

$\therefore v$

Please name the Law of Logic or Rule of Inference used in each step of your proof.

1. $p \rightarrow r$	Premise
2. $q \rightarrow s$	Premise
3. $p \lor q$	Premise
4. $r \lor s$	Rule of Constructive Dilemma with (1), (2), (3)
5. $(r \lor s) \rightarrow t$	Premise
6. t	Rule of Detachment(Modus Ponens) with (4), (5)
7. $t \rightarrow (u \land v)$	Premise
8. $u \land v$	Rule of Detachment(Modus Ponens) with (6), (7)
9. v	Rule of Conjunctive Simplification with (8)

Grading: 1 point per step with a 1 point bonus for getting everything correct.