Computer Science Foundation Exam

May 7, 2010

Section II A

DISCRETE STRUCTURES SOLUTIONS

NO books, notes, or calculators may be used,
and you must work entirely on your own.

Name: ________________________________

PID: ________________________________

In this section of the exam, there are three (3) problems.
You must do ALL of them.
They count for 40% of the Discrete Structures exam grade.
Show the steps of your work carefully.

Problems will be graded based on the completeness of the solution steps and not graded based on the answer alone.

Credit cannot be given when your results are unreadable.

Question #	Max Pts	Category	Passing	Score
1	15	PRF (Induction)	10	
2	15	PRF (Sets)	10	
3	10	PRF (Logic)	6	
ALL	40	---	26	

1) (15 pts) PRF **(**Induction)

Using induction to prove that to prove that $3 \mid (n^3 - n)$ for $n \geq 2$.

Base case: $n = 2$. $n^3 - n = 2^3 - 2 = 6$. Since $6 = 3 \times 2$, $3 \mid 6$ and the base case holds. // 2 pts

Inductive hypothesis: Assume for an arbitrary integer $n = k$ ($k \geq 2$), that $3 \mid (k^3 - k)$. // 2 pts

Inductive step: Prove for $n = k+1$ that $3 \mid ((k+1)^3 - (k+1))$ // 2 pts

$(k+1)^3 - (k+1) = k^3 + 3k^2 + 3k + 1 - k - 1$ // 3 pts
$= k^3 - k + (3k^2 + 3k)$ // 2 pts
$= 3c + (3k^2 + 3k)$, for some integer c, using the inductive hypothesis // 3 pts
$= 3(c + 3k^2 + 3k)$, proving that $3 \mid ((k+1)^3 - (k+1))$ as desired. // 1 pt

2) (15 pts) PRF **(**Sets)

(a) Prove the following for arbitrarily chosen sets *A*, *B* and *C*:

$$(A-C)-(B-C)\subseteq A-B$$

(b) Give a small example to show that these two sets, $(A-C)-(B-C)$ and $A-B$, are not necessarily equal.

(a) Let x be an arbitrarily chosen element from (A – C) – (B – C). We must prove that x is also an element of A – B.

By definition of set difference twice we have:

$x \in A \land x \notin C \land x \notin (B - C)$.
$x \in A \land x \notin C \land \neg(x \in B \land x \notin C)$.

Using DeMorgan's Law, we get:

$x \in A \land x \notin C \land (\neg(x \in B) \lor \neg(x \notin C))$, which is the same as

$x \in A \land x \notin C \land (x \notin B \lor x \in C)$, distributing, we get

$x \in A \land [(x \notin C \land x \notin B) \lor (x \notin C \land x \in C)]$, using the inverse laws we have

$x \in A \land [(x \notin C \land x \notin B) \lor F]$, and the identity law gives us

$x \in A \land x \notin C \land x \notin B$

It follows that $x \in A - B$, by definition of set difference.

// Lots of different ways to do this, grade accordingly. For the method above, there are
// about 10 steps, so one point per step.

(b) Let A = {1}, B = {} and C = {1}. Then, A – C = {}, B – C = {}, so (A – C) – (B – C) = {}, but A – B = {1}.

// 5 pts for this – 3 pts for specifying A, B and C, 2 pts for specifying (A-C)-(B-C) and A-B.

3) (10 pts) (PRF) Logic

Prove the following equivalence using the Laws of Logic only (In particular, do NOT use the Rules of Inference). Please list the rule you have used at each step.

$[(p \rightarrow r) \vee (q \rightarrow r)] \leftrightarrow [(p \wedge q) \rightarrow r]$

$(p \rightarrow r) \vee (q \rightarrow r) \leftrightarrow$	
$(\neg p \vee r) \vee (\neg q \vee r) \leftrightarrow$	Definition of Implication
$(\neg p \vee \neg q) \vee (r \vee r) \leftrightarrow$	Commutative Laws
$(\neg p \vee \neg q) \vee r \leftrightarrow$	Idempotent Laws
$\neg (p \vee q) \vee r \leftrightarrow$	De Morgan's Law
$(p \vee q) \rightarrow r$	Definition of Implication

Grading: 2 pts per step – 1 for the step, 1 for the reason