

# Computer Science Foundation Exam

August 26, 2023

## Section A

### BASIC DATA STRUCTURES

**NO books, notes, or calculators may be used,  
and you must work entirely on your own.**

### **SOLUTION**

Question #	Max Pts	Category	Score
1	10	DSN	
2	5	ALG	
3	10	ALG	
<b>TOTAL</b>	<b>25</b>	----	

**You must do all 3 problems in this section of the exam.**

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## 1) (10 pts) DSN (Dynamic Memory Management in C)

The struct `Monster_List` maintains a list of monsters using a dynamically sized array of pointers to `Monster`. A function prototype is given for a function `initializeMonster`, which takes in a pointer to a `Monster` **that must already be pointing to memory that is allocated**, and then fills that memory with information about a default monster. Write a function `getDefaultMonsters` which takes in a positive integer `n`, creates a pointer to a `Monster_List`, allocates room for it, and then fills it with `n` default Monsters, and then returns a pointer to the `Monster_list` created. (Note: You must call `initializeMonster` in your solution.)

```
typedef struct Monster {
    // Details not necessary to solve the problem.
} Monster;

typedef struct Monster_List {
    Monster** mArray;
    int numMonsters;
} Monster_List;

// Initializes the monster pointed to by mPtr to be the default
// monster.
void initializeMonster(Monster* mPtr);

Monster_List* getDefaultMonsters(int n) {

    Monster_List* res = malloc(sizeof(Monster_List)); // 2 pts

    res->mArray = malloc(sizeof(Monster*)*n); // 2 pts

    res->numMonsters = n; // 1 pt

    for (int i=0; i<n; i++) { // 1 pt
        res->mArray[i] = malloc(sizeof(Monster)); // 2 pts
        initializeMonster(res->mArray[i]); // 1 pt
    }
    return res; // 1 pt
}
```

**Grading Notes: Take off an integer number of points. For two small errors that you believe are each worth less than a point, take off 1 pt total. If there's only one tiny error (say one dot instead of arrow) correct it and give full credit.**

## 2) (5 pts) ALG (Linked Lists)

Suppose we have a singly linked list implemented with the structure below and a function that takes in the head of the list.

```
typedef struct node {
    int num;
    struct node* next;
} node;

void whatDoesItDo (node * head) {

    int tot = 0;
    while (head != NULL) {
        head->data += tot;
        tot = head->data;
        head = head->next;
    }
}
```

If we call whatDoesItDo(head) on the following list, show the list after the function has finished.

head → 3 → 9 → 7 → 1 → 4 → NULL? Please fill in the designated slots below.

→ 3 → 12 → 19 → 20 → 24 → NULL

**Grading: 1 pt per slot all or nothing, no exceptions.**

**Note:** The code is transforming the list to be a prefix sum version of the old list. (So the kth item in the update list will store the sum of all the first k items in the old list.)

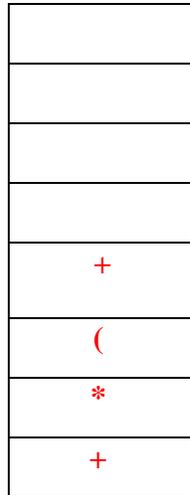
3) (10 pts) ALG (Stack)

Convert the following infix expression to postfix using a stack. Show the contents of the stack at the indicated points (A, B, and C) in the infix expression.

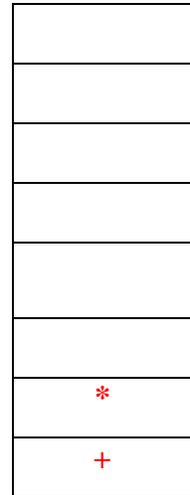
$$4 + 3 * ( 2 - 6 / ( 9 - 7 * 1 ) + ( 2 + 3 ) * 1 ) - 8 / ( 1 + 1 )$$



A



B



C

Note: A indicates the location in the expression **AFTER** the multiplication and before the open parenthesis. B indicates the location in the expression **AFTER** the addition and before the open parenthesis. C indicates the location in the expression **AFTER** the close parenthesis and before the subtraction.

Resulting postfix expression:

4	3	2	6	9	7	1	*	-	/	-	2	3	+	1	*	+	*	+	8	1	1	+	/	-
---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---

**Grading: 2 pts for each stack, 4 pts for the total expression. Give partial as necessary.**

# Computer Science Foundation Exam

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## Section B

### ADVANCED DATA STRUCTURES

**NO books, notes, or calculators may be used,  
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### **SOLUTION**

Question #	Max Pts	Category	Score
1	10	DSN	
2	10	DSN	
3	5	ALG	
<b>TOTAL</b>	<b>25</b>		

**You must do all 3 problems in this section of the exam.**

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## 1) (10 pts) DSN (Binary Trees)

Consider using the following struct definition for a node of a binary search tree:

```
typedef struct node {
    int data;
    int height;
    struct node* left;
    struct node* right;
} node;
```

Assume that a binary search tree has been built with the data values in each struct filled in, but the heights are uninitialized. Write a **void recursive** function, `assignHeights`, **with no helper** functions, which takes in a pointer, `root`, to the root of a binary search tree, and assigns the height component of each node in the subtree pointed to by `root` to its correct height in the tree. Recall that the height of a leaf node is 0, and that more generally, the height of a node is the maximum number of links (left or right) to follow from that node to any leaf node in that subtree. (If `root` is `NULL`, then no action should be taken.)

```
void assignHeights(node* root) {

    if (root != NULL) {
        assignHeights(root->left);
        assignHeights(root->right);
        int big = -1;
        if (root->left != NULL) big = root->left->height;
        if (root->right != NULL && root->right->height > big)
            big = root->right->height;
        root->height = big + 1;
    }
}
```

**Grading: 2 pts recursive call left before root height assigned (1 pt if after)**  
**2 pts recursive call right before root height assigned (1 pt if after)**  
**2 pts – attempting to determine max of left and right**  
**2 pts – avoiding NULL ptr error while getting max**  
**2 pts – setting root height to max of left and right plus 1**

## 2) (10 pts) DSN (Binary Heaps)

A **minimum heap** is typically implemented with an array, with the root node (**minimum value**) being stored in index 1 of the array. To insert a new value into a heap, it's originally placed in the first open slot, followed by running a "percolate up" operation. Write a function that inserts a value into a heap in this manner. You may assume that the array is allocated to be big enough to store the newly inserted value. The function prototype is as follows:

```
void insert(int* heap, int curSize, int newVal);
```

heap is a pointer to an array which currently stores curSize number of values (but has room for at least 1 more). newVal is the new number to be inserted into the heap. Write this function which inserts the value newVal into this **minimum heap**. Take care to avoid infinite loops or array out of bounds issues. You may assume that index curSize+1 is in bounds for the array heap. Also, remember that index 0 of the array heap is unused. **You may not write any helper functions.**

```
void insert(int* heap, int curSize, int newVal) {  
  
    heap[curSize+1] = newVal;  
    int idx = curSize+1;  
    while (idx > 1) {  
        if (heap[idx/2] < heap[idx]) break;  
        int tmp = heap[idx/2];  
        heap[idx/2] = heap[idx];  
        heap[idx] = tmp;  
        idx /= 2;  
    }  
}
```

**Grading: 1 pt place newVal in array slot curSize+1**

**1 pt – some loop or recursion**

**1 pt – access array at current index divided by 2 (parent)**

**2 pts – triggering a swap if the two appropriate items are out of order**

**3 pts – swap mechanics**

**2 pts – correctly ending the loop**

3) (5 pts) ALG (Tries)

Suppose we insert the following strings into a trie:

cantaloupe  
loop  
lop  
lobby  
cantilever

(a) How many nodes would the resulting trie contain, including the root node?

**25 (Grading: 2 pts, 1 pt for answer 23, 24,26,27)**

(b) What integer value would we store in the *count* variable in the **root node** of the resulting trie? Note that the count variable should be set to the number of valid words with end nodes stored within that subtree.

**5 (Grading: 1 pt all or nothing)**

(c) If we subsequently deleted both “lobby” and “loop” from the trie, how many total nodes would be deallocated over the course of those two deletion operations?

**5 (Grading: 2 pts, 1 pt if put 6, 7 or 8)**

# Computer Science Foundation Exam

August 27, 2023

## Section C

### ALGORITHM ANALYSIS

**NO books, notes, or calculators may be used,  
and you must work entirely on your own.**

### **SOLUTION**

Question #	Max Pts	Category	Score
1	10	ANL	
2	10	ANL	
3	5	ANL	
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## 1) (10 pts) ANL (Algorithm Analysis)

Consider the following problem:

Given two input values,  $n$  and  $k$ , determine the number of strings of length  $n$ , which only contains A's and B's, that have a run of  $k$  or more consecutive B's.

One algorithm to solve the problem is as follows:

Recursively generate each possible string of  $n$  A's and B's. These can be generated in alphabetical order, never storing more than 1 of the strings at the same time.

For each string generated, loop through the string from left to right, keeping a running tally of the current number of B's. (For example, with the string ABBABBBAAAB, the running counter would update as follows  $0 \rightarrow 1 \rightarrow 2 \rightarrow 0 \rightarrow 1 \rightarrow 2 \rightarrow 3 \rightarrow 0 \rightarrow 0 \rightarrow 1$ .) If this running tally ever equals or exceeds  $k$ , add 1 to a global counter storing the final result. For simplicities sake, assume that the loop completes going through the whole string before 1 is potentially added to the global counter.

With proof, determine the Big-Oh runtime of this algorithm in terms of the input parameter,  $n$ .

The recursive algorithm which generates each possible combination of  $n$  letters runs itself in  $O(2^n)$  time. For each letter, there are 2 choices, and we pair up each possible choice at each slot, so to get the total number of combinations we multiply 2 by itself  $n$  times to get  $2^n$  combinations. Though it's a little difficult to prove (no proof is required for full credit here), although sometimes we change more than 1 letter between combinations, over the course of the whole algorithm, the total number of letter changes does not exceed  $2^{n+1}$ , which is a constant times  $2^n$ , meaning that the original run time of  $O(2^n)$  to generate each combination is accurate.

For each combination generated, the algorithm described runs a simple loop through all the letters of the string. Since there are  $n$  letters and only a constant amount of work is done for each letter (either adding one to our running tally or setting it to 0),  $O(n)$  time is spent on each combination.

It follows that the overall running time of the algorithm is  $O(n2^n)$ .

**Grading: 5 pts for arguing that the total number of combinations is  $2^n$ .**

**3 pts for arguing that evaluating each combination takes  $O(n)$  time.**

**2 pts for concluding that the total run time is  $O(n2^n)$**

**Give partial as necessary – some credit can be given if parts of the analysis are accurate. (For example, award 2 pts out of 5 for the incorrect conclusion that there are  $n!$  valid strings of length  $n$ .)**

## 2) (10 pts) ANL (Algorithm Analysis)

A program takes  $O(n \lg m)$  time to process  $n$  data sets, each which have  $m$  values. For 100,000 data sets, each with  $2^{16}$  values, the program takes 500 ms (milliseconds) to complete. Given this information, how many milliseconds would we expect the program to take to process 60,000 data sets, each with  $2^{20}$  values?

For some constant  $c$ , let  $T(n, m) = cn \lg m$  be the run time of the algorithm for a given input size of  $n$  data sets, each with  $m$  values. Using the given information, we have:

$$\begin{aligned} T(100000, 2^{16}) &= c(100000)(\lg 2^{16}) = 500ms \\ c(100000)(16 \lg 2) &= 500ms \end{aligned}$$

$$c = \frac{500ms}{(100000)(16 \lg 2)} = \frac{1ms}{3200 \lg 2}$$

Now, using this value of  $c$ , solve for the desired information:

$$\begin{aligned} T(60000, 2^{20}) &= \frac{1ms}{3200 \lg 2} \times 60000 \times (\lg 2^{20}) \\ &= \frac{1ms \times 60000 \times 20 \times \lg 2}{3200 \times \lg 2} \\ &= \frac{1ms \times 60000}{160} = \frac{3000ms}{8} = 375ms \end{aligned}$$

**Grading: 2 pts - Setting up equation for constant**

**2 pts – Solving for  $c$  (no simplification necessary)**

**2 pts – Plugging in appropriate new values to solve problem**

**4 pts – arriving at correct answer in ms (give partial as necessary)**

## 3) (5 pts) ANL (Summations)

Solve the summation below. Your final result should be a function in terms of  $n$ .

$$\sum_{k=1}^{2n} \left( \frac{k}{2} + 3 \right)$$

$$\sum_{k=1}^{2n} \left( \frac{k}{2} + 3 \right) = \left( \frac{1}{2} \sum_{k=1}^{2n} k \right) + \left( \sum_{k=1}^{2n} 3 \right)$$

$$= \frac{1}{2} \times \frac{2n(2n+1)}{2} + 3(2n)$$

$$= \frac{n(2n+1)}{2} + \frac{12n}{2}$$

$$= \frac{2n^2 + n + 12n}{2}$$

$$= n^2 + \frac{13}{2}n$$

**Grading: 1 pt – split sum**

**2 pts – correct plug in for formula sum of k**

**2 pts – simplify to correct answer (give 1 pt if final form isn't either factored or polynomial form but if progress is made)**

# Computer Science Foundation Exam

August 26, 2023

Section D

ALGORITHMS

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**SOLUTION**

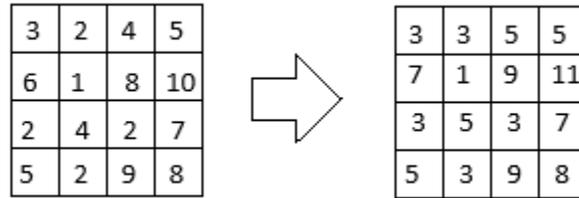
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1) (10 pts) DSN (Recursive Coding)

Imagine that a virus is spreading through an area we can model as a 2-dimensional integer array of size  $n$  by  $n$ , where each value in the array stores the current level of virus. The virus can be activated at a particular location. If that location currently has a virus level that is an even integer, then the virus level in that square increases by 1, and then the activation recursively activates the locations above, below, left and right of the immediate location. If the virus level in that square was odd, no change occurs and the virus doesn't spread. Luckily, once the virus level in a square increases by 1 due to an activation, it can't increase again. Here is a small example of a before and after picture of activating the virus in row 2, column 0:



Complete the **recursive** function below so that it takes in a pointer to the array storing the grid, the value of  $n$ , and the row and column of where the virus is activated, and updates the virus levels accordingly. Don't forget to make sure you don't go out of bounds of the array!

```
void activate(int** grid, int n, int row, int col) {
    if (row < 0 || row >= n) return;

    if ( col < 0 || col >= n ) return; // col out of bounds

    if ( grid[row][col]%2 == 1 ) return; // odd square

    grid[row][col]++;

    activate(grid, n, row-1, col) ;

    activate(grid, n, row, col-1) ;

    activate(grid, n, row, col+1) ;

    activate(grid, n, row+1, col) ;

}
```

- Grading: 2 pts out of bounds check for col**
- 3 pts for odd square check**
- 1 pts for incrementing correct item**
- 1 pt each recursive call, give credit if last two parameters are correct (order doesn't matter)**

## 2) (5 pts) ALG (Sorting)

Show the result after each iteration of performing Insertion Sort on the array shown below. For convenience, the result after the first and last iterations are provided. The first row of the table contains the original values of the array.

Iteration	Index 0	Index 1	Index 2	Index 3	Index 4	Index 5	Index 6	Index 7
0	16	3	18	15	1	8	12	4
1	3	16	18	15	1	8	12	4
2	3	16	18	15	1	8	12	4
3	3	15	16	18	1	8	12	4
4	1	3	15	16	18	8	12	4
5	1	3	8	15	16	18	12	4
6	1	3	8	12	15	16	18	4
7	1	3	4	8	12	15	16	18

**Grading: 1 pt per row, row has to be perfectly correct to get the point.**

## 3) (10 pts) ALG (Base Conversion)

Perform each of the requested base conversions (the base of each number is written as a subscript):

( a ) (2 pts)  $347_{10} = \underline{2342}_5$

$$\begin{array}{r} 5 \mid 347 \\ 5 \mid 69 \text{ R } 2 \\ 5 \mid 13 \text{ R } 4 \\ 5 \mid 2 \text{ R } 3 \end{array}$$

**Grading: 1 pt for answer, 1 pt for process**

( b ) (2 pts)  $361_7 = \underline{190}_{10}$

$$361_7 = 3 \times 7^2 + 6 \times 7^1 + 1 = 3 \times 49 + 42 + 1 = 147 + 42 + 1 = 190$$

**Grading: 1 pt for answer, 1 pt for process**

( c ) (3 pts)  $3AD_{16} = \underline{001110101101}_2$

$3AD_{16} = 0011\ 1010\ 1101_2$ , since 16 is a perfect power of 2, we can convert each hex character to exactly 4 bits. These conversions are common enough most CS students can be expected to have them memorized, so there's no need to show work except for writing the answer down.

**Grading: Based on answer only, 1 pt for first 4 bits, 1 pt for next for bits, 1 pt for last four bits. All 4 bits have to be correct in the group to get the point.**

**Give full credit for any valid process ( 16 → 10 → 2 is harder but valid)**

( d ) (3 pts)  $247321_8 = \underline{110323101}_4$

$247321_8 = 010\ 100\ 111\ 011\ 010\ 001_2$  (As previously mentioned, we can convert to base 2 in this manner since 8 is a perfect power of 2).

Now, we can use similar thinking to convert from base 2 to base 4. Just remember to grab bits from the right to the left:

$$010\ 100\ 111\ 011\ 010\ 001_2 = 01\ 01\ 00\ 11\ 10\ 11\ 01\ 00\ 01_2 = 110323101_4$$

**Grading: 1 pt for intermediate step to base 2, 2 pts for final answer**

**Give full credit for any valid process ( 8 → 10 → 4 is harder but valid)**

**Note: Leading 0s are not necessary for full credit.**