Computer Science Foundation Exam

August 6, 2001

Section I A

No Calculators!

Name: _____

SSN: _____

In this section of the exam, there are three (3) problems.

You must do all of them.

The weight of each problem in this section is indicated with the problem.

The algorithms in this exam are written in a combination of pseudocode and

programming language notation.

Partial credit can not be given unless all work is shown.

As always, be complete, yet concise, and above all <u>be neat</u>,

credit can not be given when your results are unreadable.

(1, 20%) Given the following array of integers and algorithm, answer the questions below. Assume that the global array **A[1..n]** is correctly declared and contains the values shown.

Array A	1	4	2	5	8	6
position	1	2	3	4	5	6

```
algorithm P1
  x, y, i, j, k : integer

  i ← 1
  j ← 6
  loop
    k ← i
    x ← A[i]
    y ← A[j]
     loop
        exitif (k > j)
        if(A[k] > x) then
           x ← A[k]
        endif
        if(A[k] < y) then
           y ← A[k]
        endif
        k ← k + 1
     endloop
     A[i] ← x - y
     A[j] ← 2 * y
     i ← i + 1
     j ← j - 1
     exitif (i > j)
  endloop
endalgorithm
```
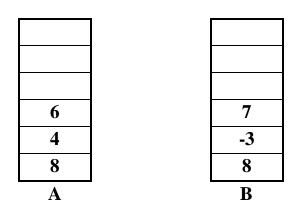
a) Show the array **A** after the algorithm has completed execution (2pts each)?

Array A	7	6	3	4	4	2
position	1	2	3	4	5	6

b) What value will the variables **x** and **y** contain when the algorithm is finished (4pts each)?

x	5	y	2

(2, 16%) In the following Postfix expressions all values are single decimal digits and the operations are addition "+", subtraction "−", multiplication "*" and division "/". In each box below the Postfix expression in part a) , show ONLY the contents of the stack at the indicated point in the Postfix string (point A or B). Put the final answer in the blank. If the Postfix string is invalid, carry the operations as far as possible and write "invalid" as the answer. (4 points total)

a)   8 4 3 2 * ^{A} - 6 * 4 / 7 ^{B} - +   = **-2**

6	**7**
4	**-3**
8	**8**
A	**B**

Assume that you have a stack and a queue and the standard stack and queue operations: `push, enqueue, top, dequeue and pop.` Execute, in top-to-bottom order, the operations below and answer questions **b)**, **c)** and **d)** below. (12 points total)

```
enqueue(4)
push(3)
enqueue(7)
push(6)
enqueue(pop)
print(dequeue)
push(dequeue)
enqueue(2)
push(5)
enqueue(pop)
print(pop)
print(dequeue)
print(pop)
```

b) show the output from the print statements:

4	**7**	**6**	**3**
first output	second output	third output	fourth output

c)   After the above operations were completed, was the stack empty?   **Yes**

d) After the above operations were completed, was the queue empty? **<u>No</u>**

(3, 18%) Answer each of the following "timing" questions concerning an algorithm of a particular order and a data set of a particular size. Assume that the run time is affected only by the size of the data set and not its composition and that **n** is an arbitrary integer.

a) For an **O(log₂n)** algorithm, one data set with **n = 128** takes **24.5** seconds.

How long will it take for a data set with **n = 8**? (4 pts) <u>**10.5 sec**</u>

$$\frac{\log_2(128)}{24.5} = \frac{\log_2(8)}{x} \qquad\qquad \frac{7}{24.5} = \frac{3}{x} \qquad\qquad 3(24.5) = 7x$$

7x = 73.5 **x = 10.5**

b) For an **O(n³)** algorithm, one data set with **n = 3** takes **54** seconds.

How long will it take for a data set with **n = 5**? (4 pts) <u>**250 sec**</u>

$$\frac{3^3}{54} = \frac{5^3}{x} \qquad \frac{27}{54} = \frac{125}{x} \qquad\qquad \frac{1}{2} = \frac{125}{x} \qquad x = 2(125) \qquad x = 250$$

c) For an **O(2ⁿ)** algorithm, a friend tells you that it took **17** seconds to run on her data set. You run the same program, on the same machine, and your data set with **n = 7** takes **68** seconds.

What size was her data set? (4 pts) <u>**n = 5**</u>

$$\frac{2^x}{17} = \frac{2^7}{68} \qquad\qquad \frac{2^x}{17} = \frac{128}{68} \qquad 2^x(68) = 128(17) \qquad 2^x(4) = 128(1)$$

2ˣ = 128 / 4 = 32 thus x = 5

Given the following pseudocode segment, answer the questions below for an arbitrary **n**:

```
x ← 0
for i ← 1 to (2*n) do
     for j ← 1 to i do
          x ← x + 3
```

d) What is the Order of this pseudocode segment? (3 pts) <u>**O(n²)**</u>

e) What will be the value of **x** when the **for** loops end? (3 pts)

$$\sum_{i=1}^{2n}\sum_{j=1}^{i}3 = 3\sum_{i=1}^{2n}\sum_{j=1}^{i}1 = 3\sum_{i=1}^{2n}i = 3\frac{2n(2n+1)}{2} = 3\left(n(2n+1)\right) = \underline{6n^2 + 3n}$$

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