Computer Science Foundation Exam

May 6, 2005

Computer Science

Section 1B

No Calculators!

KEY

Name: ______________________________

SSN: ______________________________

Score: / **50**

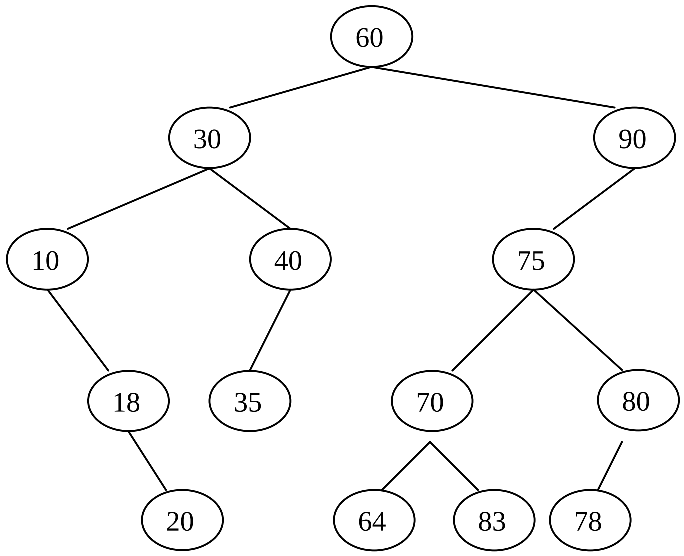
In this section of the exam, there are four (4) problems. You must do all of them.

The weight of each problem in this section is indicated with the problem. Partial credit cannot be given unless all work is shown and is readable.

Be complete, yet concise, and above all be neat.

1. [15 points]

Given the binary tree shown below, answer questions (a) and (b).



(a – 3 points) Is the binary tree shown above a valid binary search tree? YES NO
Circle the correct answer.

(b – 12 points) Determine the order in which the nodes of the binary tree shown above are visited assuming the function **A(root)** is invoked. Assume that the tree nodes and pointers are defined as shown. Assume that **root** is a pointer to the node containing 60.

```
struct treeNode{
   int data;
   struct treeNode *left, *right:
}
struct treeNode *tree_ptr;

void A(struct tree_ptr *node_ptr){
    if (node_ptr != NULL){
       printf(“%d ,”,node_ptr->data);
       B(node_ptr->left);
       B(node_ptr->right);
    }
}

void B(struct tree_ptr *node_ptr){
    if (node_ptr != NULL) {
       A(node_ptr->left);
       printf(“%d ,”,node_ptr->data);
       A(node_ptr->right);
   }
}
```

Solution to Question 1 part b:

60	**10**	**18**	**20**	**30**	**40**	**35**	**75**	**64**	**70**	**83**	**78**	**80**	**90**

Write a recursive function that will find the height of a binary tree. The height of an empty tree is defined as zero. The height of a single node tree is defined as 1. You may assume the same tree node structure as given in problem #1.

One possible solution is:

```
int height (struct treeNode *ptr)
{
     int leftheight, rightheight;

     if (ptr == NULL)
     return 0;
     else
     {     leftheight = height(ptr->left);
           rightheight = height(ptr->right);
           if (leftheight > rightheight)
               return(leftheight + 1);
           else
               return(rightheight + 1);
     }
}
```

3. [15 points – 5 points each]

Find the closed form expression in terms of the parameter **n** or an exact value if the summation limits are known, for each of the following summations.

Show all of your work…an answer alone is not sufficient to receive full credit.

(a) (5 points) Find the closed form for: $\sum_{j=8}^{34}(4j+3)$

$\boxed{\text{Closed form} = 2349}$

$$\sum_{j=8}^{34}(4j+3)=\sum_{j=1}^{34}(4j+3)-\sum_{j=1}^{7}(4j+3)=\frac{4(34)(35)}{2}+3(34)-\frac{4(7)(8)}{2}-3(7)=$$

$$=2380+102-112-21=2349$$

(b) (5 points) Find the closed form for: $\sum_{i=1}^{3n+2}(5i-4)$

$\boxed{\text{Closed form} = \frac{45n^2+51n+14}{2}}$

$$\sum_{i=1}^{3n+2}(5i-4)=5\sum_{i=1}^{3n+2}i-4\sum_{i=1}^{3n+2}1=\frac{5(3n+2)(3n+3)}{2}-4(3n+2)=\frac{5(9n^2+15n+6)}{2}-4(3n+2)$$

$$=\frac{45n^2+75n+30-24n-16}{2}=\frac{45n^2+51n+14}{2}$$

(c) (5 points) Find the closed form for: $\sum_{i=0}^{3n}(4i-2j)$

$\boxed{\text{Closed form} = 18n^2+6n-6jn-2j}$

$$\sum_{i=0}^{3n}(4i-2j)=4\sum_{i=0}^{3n}i-2j\sum_{i=0}^{3n}1=\frac{4(3n)(3n+1)}{2}-2j(3n+1)=\frac{36n^2+12n}{2}-6jn-2j$$

$$=18n^2+6n-6jn-2j$$

4. **[5 points – 2 points part (1), 3 points part (b)]**
Perform the specified number conversion in parts (a) and (b) below. Show all of your work. An answer alone will not receive full credit.

(a) What is the decimal equivalent to the binary number: 11101110
Show all work.

answer:

238 since,
$(1\times 2^7) + (1\times 2^6) + (1\times 2^5) + (1\times 2^3) + (1\times 2^2) + (1\times 2^1)$
$= 128 + 64 + 32 + 8 + 4 + 2$
$= 238$

(b) What is the binary equivalent to the decimal number: 136.87

answer:

Integer part:
10001000 since,
136/2 = 68 remainder 0 (LSB)
68/2 = 34 remainder 0
34/2 = 17 remainder 0
17/2 = 8 remainder 1
8/2 = 4 remainder 0
4/2 = 2 remainder 0
2/2 = 1 remainder 0
1/2 = 0 remainder 1 (MSB)
reading from MSB to LSB (bottom to top) gives 10001000

Fractional part:
11011110… since
$.87 \times 2 = .74$ carry of 1 (MSB)
$.74 \times 2 = .48$ carry of 1
$.48 \times 2 = .96$ carry of 0
$.96 \times 2 = .92$ carry of 1
$.92\times 2 = .84$ carry of 1
$.84\times 2 = .68$ carry of 1
$.68 \times 2 = .36$ carry of 1
$.36 \times 2 = .72$ carry of 0
$.72 \times 2 = .44$ carry of 1
$.44 \times 2 = .88$ carry of 0
$.88 \times 2 = .76$ carry of 1 … gives **.1101111**
…

10001000.110111