Computer Science Foundation Exam

March 3, 2000

Section I A

No Calculators!

Name: _____

SSN: _____

In this section of the exam, there are three (3) problems.

You must do all of them.

The weight of each problem in this section is indicated with the problem.

The algorithms in this exam are written in a combination of pseudocode, and programming language notation.

Partial credit can not be given unless all work is shown.

If you need extra room to do work to be graded then do so on the last page attached to this test. Make sure to clearly label the problem you are working on.

As always, be complete, yet concise, and above all <u>be neat</u>,

credit can not be given when your results are unreadable.

(1, 20%) Given the following array of numbers and procedure, answer the questions below. Assume that the global array **X[1..n]** is correctly declared and contains the values shown. (So n=6.)

Array X	**1**	**3**	**1**	**2**	**1**	**2**
Position	1	2	3	4	5	6

```
procedure ArrayOperation()
   i, j, k, s, t: integer;
   i ← 0;
   while (i < n) do
        i ← i + 1;
        s ← 0;
        j ← 1;
        while (j <= n) do
             s ← X[j] + s;
             j ← j + 1;
        endwhile
        X[i] ← s;
   endwhile
endprocedure
```

a) (6 pts) Show the array **X** after the procedure has executed?

Array X	**10**	**19**	**35**	**69**	**136**	**271**
Position	1	2	3	4	5	6

b) (7 pts) Consider the case where each element of the array X is equal to m, where m is a positive integer. Now consider executing the procedure ArrayOperation(). What will the sum of the elements of the array X be, in terms of m?

6m + 11m + 21m + 41m + 81m + 161m = **321m**

c) (7 pts) Consider the case where each element of the array X is a positive integer. If X[i] = X[i+1] for some integer i such that $0 < i < 6$, what can we say about the relationship between X[i] and X[i+1] AFTER executing the procedure ArrayOperation()?

X[i+1] will be equal to 2*X[i] minus the old value of X[i+1].

(2, 18%) The following are Postfix expressions. All values are single decimal digits and the operations are addition "+", subtraction "–", multiplication "*" and division "/". (Note that the final answer and intermediate answers in the stack may not be single decimal digits.) In each box below the Postfix expression, show ONLY the contents of the stack at the indicated point in the Postfix string (point A, B or C). Put the final answer in the blank. If the Postfix string is invalid, carry the operations as far as possible and write "invalid" as the answer. (6 points each)

a) **3 8 * 2 4 A + / 1 3 - B / 9 5 * C + = 43**

A	B	C
4		
2	-2	45
24	4	-2

b) **2 6 1 4 * A - 8 * 4 B 2 1 * / C / * = 16**

A	B	C
4	4	2
6	16	16
2	2	2

Next to each Postfix expression, circle one answer to indicate if it is a valid Postfix string or not: (no extra credit for providing the answer, if it is valid) (3 points each)

c) **9 5 3 7 2 8 2 * + / - * + -** **Invalid**

d) **6 4 + 8 * 3 - 2 + 3 *** **Valid**

(3, 20%) Answer each of the following "timing" questions concerning an algorithm of a particular order and a data instance of a particular size. Assume that the run time is affected by the size of the data instance and not its composition.

a) (4 pts) For an **$O(n\log_2 n)$** algorithm, an instance with **n = 16** takes **100** seconds.

How long will it take with **n = 32**? **250 seconds**

b) (4 pts) For an **$O(n^3)$** algorithm, an instance with **n = 512** takes **56** milliseconds.

If you used a different-sized data instance and it took **7** milliseconds how large must that instance be? **256**

c) (4 pts) For an **$O(n^k)$** algorithm, where k is a positive rational number, a friend tells you that her instance of size m took **16** seconds to run. You run an instance of size 4m and find that it takes **256** seconds to run. What is the value of k? **k=2**

Given the following pseudocode segment, answer the questions below for an arbitrary positive even integer **n**:

```
x ← 0
for i ← 1 to (n*(8*n+8)) do
     for j ← n/2 to n do
        x ← x + (n - j)
```

d) (2 pts) What is the Order of this code segment, in terms of n? **$O(n^3)$**

e) (6 pts) What will be the value of **x** (in terms of n) when the **for** loops end? **$n^2(n+1)(n+2)$**

Extra Work Page - Please clearly label any work on this page that you would like graded.

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March 3, 2000

Section I B

No Calculators!

Name: _____

SSN: _____

In this section of the exam, there are three (3) problems.

You must do all of them.

The weight of each problem in this section is indicated with the problem.

The algorithms in this exam are written in a combination of pseudocode, and programming language notation. The algorithms that you are asked to produce should use a syntax that is clear and unambiguous.

Partial credit can not be given unless all work is shown.

If you need extra room to do work to be graded then do so on the last page attached to this test. Make sure to clearly label the problem you are working on.

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(4, 12%) Given a global array of numbers **X[1..n]**, you are to write a recursive function **SumEvenArray** that will return the sum of the even array elements in the range specified by the parameters to the function. Assume that the array was already initialized and that the **SumEvenArray** function will be called as shown below and answer will be equal to the sum of the even elements in the array X in between the indices of i and j, inclusive. (Assume that i <= j for each function call. Also, if there are no even elements in the array in the specified range, your function should return 0.)

answer ← SumEvenArray(i,j)

In the space below, write a RECURSIVE function **SumEvenArray.**

```
SumEvenArray(i, j isoftype in Num)
     value isoftype Num
     value <- 0
     if (X[i] MOD 2 = 0) then
          value <- X[i]
     endif

     if (i = j) then
          return value
     else
          return value + SumEvenArray(i+1,j)
     endif
endfunction
```

(5, 18%) Find the closed form or exact value for the following:
(n is an arbitrary positive integer; for part b assume n>2):

a) (6 pts) $\sum_{i=1}^{50}(4ni-5)$ = **5100n - 250**

b) (6 pts) $\sum_{i=7}^{2n+1}(2i+6)$ = **2(n+7)(2n-5)**

c) (6 pts) $\sum_{i=21}^{40}(10i-25)$ = **5600**

(6, 12%)

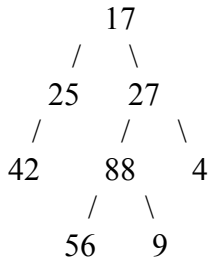
a) (8 pts) Consider the following record type definition for a binary tree node:

```
Tree_Node definesa record
      data isoftype Num
      left_child isoftype Ptr toa Tree_Node
      right_child isoftype Ptr toa Tree_Node
   endrecord
```

Now, consider the following procedure that traverses a binary tree and prints out the value of the number stored at each node in the tree.

```
Procedure Bin_Tree_Trav (current_ptr isoftype in Ptr toa Tree_Node)
   if  (current_ptr <> NIL) then
         Bin_Tree_Trav(current_ptr^.right_child)
         print(current_ptr^.data)
         Bin_Tree_Trav(current_ptr^.left_child)
   endif
endprocedure
```

Assume that head is a Ptr toa Tree_Node. More specifically, head is pointing to the head of the tree below.



What would the procedure call **Bin_Tree_Trav(head)** print out?

4 , 27, 9, 88, 56, 17, 25, 42

b) (4 pts) Given a linked list storing n numbers, what is the average number of elements compared (in terms of n) to search for a value in the list if 50% of the values searched for are not in the linked list. (Assume that the list is not ordered in any particular manner and that values found in the list are randomly located.)

¾*n

Extra Work Page - Please clearly label any work on this page that you would like graded.