Designing Simulated Learners that Externalize Metacognition and Self-regulation for Complex Problem Solving

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Abstract. High school students consistently struggle with algebra problem-solving, not solely because of computational deficits but primarily because of inadequate metacognitive and self-regulatory strategies. Current intelligent tutoring systems, while demonstrating effectiveness in procedural skill development, frequently fail to adequately support the development of such higher-order thinking processes. This paper examines the cognitive and metacognitive challenges inherent in algebra word-problem solving and presents an initial theoretical and empirically driven design framework for simulated learners as pedagogical agents that model, teach, learn, externalize, and explain metacognitive processes embedded in *MetaSim*, a new interactive learning environment (ILE) for algebra problem-solving. By making invisible metacognitive processes visible through simulated peer learners who demonstrate both productive and unproductive strategies, students can develop more sophisticated approaches to complex mathematical problem-solving. This paper describes thirteen specific design principles grounded in theories of metacognition and self-regulated learning to guide the design of simulated learners in ILEs.

Keywords: Simulated Learners · Metacognition · Self-Regulated Learning · Design Principles · Complex Problem Solving · Algebra

1 Introduction

Algebra is a critical gateway to advanced mathematical thinking and a foundational requirement for success in STEM fields. Despite its importance, algebra remains one of the most challenging subjects for high school students, with failure rates consistently higher than in other mathematical domains. According to the most recent NAEP [1] results, U.S. high school mathematics achievement is at a historically low point. In 2024, the average 12th-grade math score declined further from 2019 levels, with only about 22% of seniors performing at or above the NAEP Proficient level and roughly 45% scoring below Basic, the highest proportion ever recorded. Although NAEP does not report algebra as a standalone score, it accounts for roughly one-third of the Grade

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12 math assessment, suggesting that the overall decline reflect substantial difficulties with algebraic reasoning, functions, and problem-solving. Together, the results indicate that many high school students are leaving school without the foundational algebra skills needed for college and career readiness.

While traditional explanations have focused on computational difficulties or conceptual misunderstandings, emerging research suggests that the primary obstacle lies not in what students know but in how they regulate and monitor their thinking during problem-solving [2]. Students who struggle with algebra often possess relevant procedural knowledge but fail to deploy metacognitive strategies such as planning, monitoring progress, evaluating solution paths, and reflecting on errors [3].

Current intelligent tutoring systems have made remarkable strides in personalizing instruction and providing immediate feedback on procedural steps [4]. However, these systems typically emphasize correct solution pathways while providing limited support for the metacognitive dimension of learning [5]. Students may successfully complete problems in the tutoring environment yet fail to transfer these skills to novel contexts because they have not developed the self-regulatory competencies necessary for independent problem-solving. This gap between procedural proficiency and metacognitive sophistication represents a fundamental limitation in current educational technology for learning, transfer and independent problem solving.

This paper makes two contributions to address these limitations. First, it proposes simulated learners as a promising pedagogical approach [6]. Simulated learners are computer-generated agents that model realistic learning behaviors, including both productive and unproductive problem-solving strategies, while externalizing the typically invisible metacognitive processes involved in algebraic thinking. By observing, critiquing, and teaching human learners, students engage in metacognitive reflection without the performance pressure of their own problem-solving. Second, this paper presents an initial set of design principles for designing simulated learners in *MetaSim*, a new ILE for algebra word-problem solving and metacognitive SRL skills.

2 Metacognition and Self-Regulated Learning (SRL)

Metacognition and self-regulated learning (SRL) involve processes that unfold dynamically over time [7–11]. Learners navigate learning environments by cycling through phases of awareness, monitoring, regulation, evaluation, reflection, and adaptation. First, they develop awareness of task demands and their own cognitive resources. Then, learners monitor their progress, emotions, motivation, and comprehension as learning unfolds in real time. This monitoring leads to regulatory actions, such as modifying strategies, managing emotions, or renewing effort. These actions address discrepancies between current performance and goals. After completing tasks or at key intervals, learners evaluate the effectiveness of their strategies and outcomes. They reflect on what worked, what didn't, and why, considering how strategies interacted with emotions and motivation. This reflection guides adaptive planning. Learners modify their plans, update their self-perceptions, readjust their goals, and refine their metacognitive knowledge and skills for future learning. Each learning experience informs and transforms the next. This dynamic process unfolds as a recurring cycle of cognitive execution, metacognitive monitoring, affective experiences, and motivational beliefs, which can be induced, disrupted, augmented, taught, modeled, explained, facilitated, supported, or fostered by simulated learners. Before we discuss design principles, we will briefly provide an overview of the metacognitive and SRL challenges experienced by high-schoolers with algebra word-problem solving.

2.1 Metacognitive and SRL Challenges in Algebra Word-Problem Solving Among High School Students

There is an extensive literature indicating that high school students struggle with metacognitive knowledge and regulation in algebra, which prevents mathematical success [3, 12]. For example, many learners possess limited awareness of their own cognitive processes when solving algebraic equations. Students often lack declarative knowledge of effective problem-solving strategies, failing to recognize when factoring is more appropriate than substitution, or when graphical approaches can clarify relationships that symbolic manipulation obscures [13]. This knowledge gap extends to conditional knowledge when students may memorize procedures without understanding the contextual appropriateness of different methods.

A learner's metacognitive accuracy presents another obstacle. High schoolers frequently exhibit either overconfidence or, conversely, mathematical anxiety, which distorts their self-assessment accuracy [14]. A learner might believe they understand quadratic equations after superficial practice, only to falter when problems require deeper conceptual transfer. This miscalibration between perceived and actual competence undermines effective study habits and help-seeking behaviors [15].

The regulatory dimension is also challenging. In the planning phase, learners may rush into calculations without analyzing the problem structure. They may not select appropriate strategies [16]. Many fail to use relevant prior knowledge or break complex, multi-step problems into smaller parts [17]. Weaknesses become apparent as students proceed through solutions without checking intermediate steps for reasonableness. Many lack the habit of asking themselves whether their algebraic manipulations preserve equality or whether the obtained solutions satisfy the original constraints [18]. This failure to engage in ongoing self-assessment means errors propagate unchecked through lengthy solution processes.

Evaluation difficulties happen at task completion. Learners rarely verify answers by substitution or other methods. They may accept incorrect results without reflection [19]. When they see incorrect solutions, they often cannot find where their reasoning went wrong. They may blame failure on their own ability rather than on strategic errors [20]. Knowledge and regulation interact, creating additional difficulties. Without knowing multiple solution paths, students cannot monitor if their approach is effective. Without monitoring, they cannot recognize when to switch strategies. These deficits are interconnected, illustrating why metacognitive development is crucial in algebra education.

3 Metacognitive and SRL-Based Design Principles for Simulated Learners in Algebra Interactive Learning Environments (ILEs)

In this section, we present thirteen theoretically and empirically based design principles derived from the literature in educational, cognitive, and instructional psychology, learning sciences, and AI in education, focused on the metacognitive, self-regulated learning, and performance aspects of learner-simulated learning interaction in *MetaSim* (see Table 1). We acknowledge other relevant contributions to the design of *MetaSim*, including cognitive [21, 22] and metacognitive architectures [23], learning theories (e.g., human and computerized tutoring [24–26]); cognitive load theory [27]; cognitive flexibility theory [28], and extensive literature on math and algebra problem-solving and tutoring (e.g., [29]) that will not be presented in this paper due to space limitations.

Table 1. List of Theory and Empirical-based Principles about Metacognition, SRL, and Problem Solving Underlying the Design of *MetaSim*'s Simulated Learners.

1	Externalize Metacognitive Processes and Skills Explicitly
2	Strategic Diversity and Flexibility
3	Scaffolded Metacognitive Prompting
4	Error Detection and Correction Modeling
5	Goal-Oriented Planning and Decomposition
6	Calibrated Self-Assessment and Help-Seeking
7	Dialogic Interaction and Reciprocal Teaching
8	Domain-Specific Metacognitive Strategies
9	Affective and Motivational Authenticity
10	Formative Feedback with Metacognitive Reflection
11	Progressive Complexity and Challenge Adaptation
12	Context-Rich and Meaningful Problem Situations
13	Bidirectional Learner-Simulated Learner Adaptation using Multimodal Data

3.1 Principle 1: Externalize Metacognitive Processes and Skills Explicitly

Effective metacognitive instruction requires making learners' internal thought processes visible. This practice is central to cognitive apprenticeship models. In these models, experts externalize their reasoning for novices to observe and internalize [30]. Research shows that students benefit from observing both expert and novice reasoning. Externalization can be achieved through verbal protocols and think-aloud procedures [31]. Simulated learners must make their metacognitive thinking audible or visible using think-aloud protocols, reflective statements, or visualizations of cognitive processes [32]. These externalizations should include planning statements before solving problems. For example: “Before I start solving, let me make sure I understand what this problem is asking.” They should include monitoring statements during problem-solving. For instance: “That answer doesn’t seem reasonable for the context of this problem, so I should check my work.” Evaluative, reflective statements after solving or changing an approach are also important. For example: “I tried using the distributive property, but I’m getting confused, so maybe I should try a different approach.” Such statements model the internal dialogue of effective self-regulated learners but are lacking in struggling students.

The system should display thought bubbles or dialogue boxes that show the simulated learner’s internal reasoning during problem-solving. This makes metacognitive processes, which are typically invisible, into observable phenomena. Explicit externalization may use both static and dynamic representations. Examples include problem-solving formulas and diagrams, paired with metacognitive strategies. It may also include gestures and verbalization by simulated learners. A skill-o-meter can indicate proficiency in specific metacognitive self-regulated learning skills [33]. To increase cognitive transparency, designers should include simulated learners who verbally describe, explain, and demonstrate both correct and flawed reasoning. This allows students to observe realistic cognitive processes at work. Learning from errors requires how mistakes happen and how to identify and correct them (Siegler, 2002). The environment should offer simulated learners at multiple difficulty levels, including struggling, developing, and proficient. These models can show different metacognitive competencies and help students recognize their own developmental stage.

Effective externalization demands careful attention to detail. The language ought to be accessible to the target student population while modeling sophisticated metacognitive awareness. Achieving this balance is challenging. “Metacognitive talk” can be abstract and complex. Such talk calls for sophisticated language and assumes learners have reached certain cognitive and developmental stages (e.g., theory of mind; [34]). If externalizations are too sparse, they may not make critical processes visible enough. If too detailed, they could seem overwhelming or unnatural. The optimal level will likely vary based on a student’s developmental level and individual differences, such as prior knowledge or working memory. This balance should be tested and validated with learners.

3.2 Principle 2: Strategic Diversity and Flexibility

Exposure to multiple solution strategies promotes adaptive expertise and helps students develop conditional knowledge about when different approaches are most appropriate for particular problem types [35]. To support this, students need to understand that mathematical problems often have multiple valid solution pathways, and flexibility in strategy selection distinguishes expert from novice problem solvers [36]. Building on the idea of explicit modeling, simulated learners should be programmed to employ varied algebraic strategies, including graphical, symbolic, numerical, and verbal approaches, demonstrating that mathematical thinking encompasses more than procedural execution. These simulated learners should demonstrate strategy selection based on problem characteristics, offering explanations such as “This equation has small coefficients, so I’ll use factoring rather than the quadratic formula” to help students understand strategic decision-making. The system should include explicit comparisons between strategies, highlighting efficiency trade-offs and helping students develop a nuanced understanding of when particular methods are advantageous. To reinforce flexibility, simulated learners should model adaptive strategy switching when initial approaches prove unproductive, demonstrating the metacognitive skill of recognizing when to abandon ineffective methods and try alternative approaches [13].

3.3 Principle 3: Scaffolded Metacognitive Prompting

Direct instruction in metacognitive strategies significantly improves mathematical problem-solving performance, particularly when instruction includes explicit modeling and practice opportunities [3]. Building from strategic diversity, scaffolding should be gradually faded as learners develop independence and internalize self-regulatory processes, a principle supported by extensive research on instructional support [37]. Simulated learners should model self-questioning techniques that promote planning, monitoring, and evaluation, asking questions like "What is this problem asking?" "What do I already know that might help?" and "Does this answer make sense given the problem context?" The environment should include prompts that encourage students to evaluate the simulated learner's metacognitive decisions, fostering critical analysis of the effectiveness of strategies. To ensure students transfer responsibility, designers should implement adaptive fading mechanisms that initially prompt extensive metacognitive self-talk, which progressively decreases as students demonstrate increasing competence, transferring regulatory responsibility from the simulated learner to the student. The system should provide metacognitive checklists or frameworks that simulated learners explicitly reference, making these tools available for students to adopt in their own problem-solving processes [38].

3.4 Principle 4: Error Detection and Correction Modeling

Learning from errors requires metacognitive awareness to detect mistakes during problem-solving and strategic knowledge to remediate them effectively [39, 40]. Students benefit from explicit error analysis and correction modeling, as these processes help them develop monitoring skills that prevent error propagation [19]. Simulated learners should make authentic algebraic errors that students can identify and correct, providing opportunities for diagnostic reasoning about mathematical misconceptions. These simulated learners should model self-monitoring behaviors, such as checking work at intermediate steps, verifying solutions by substituting them into the original equations, and testing the reasonableness of answers against problem constraints. Simulated learners should demonstrate recovery strategies when errors are detected mid-solution, showing students that mistakes are typical during of problem-solving and that effective learners have strategies for getting back on track. The simulated learners should include reflective statements such as "Wait, this doesn't look right, let me check my algebra from the previous step" to explicitly model monitoring processes. The system should require students to diagnose why simulated learners made specific errors, promoting evaluation skills and deepening understanding of common algebraic misconceptions.

3.5 Principle 5: Goal-Oriented Planning and Decomposition

Successful problem solvers engage in planning, preliminary analysis, and goal decomposition before executing solutions, a finding consistently demonstrated across studies of expert-novice differences [41]. Planning behaviors distinguish expert from novice problem solvers, with experts spending proportionally more time analyzing problems before attempting solutions [13]. Simulated learners should explicitly articulate problem goals before solving, verbalizing what they are trying to find and what information is provided. These simulated learners should model decomposition of multi-step problems into manageable sub-goals, showing students how complex problems can be broken down into sequences of simpler tasks. Simulated learners should demonstrate backward reasoning from desired outcomes, asking "What would I need to know to find this?" and working backward to identify necessary intermediate steps. The system should show planning phases in which simulated learners identify known and unknown variables, relevant formulas and relationships, and the necessary steps to reach the solution. Simulated learners should include visual representations such as diagrams, tables, and concept maps that they create during planning, demonstrating how external representations support problem analysis and solution design.

3.6 Principle 6: Calibrated Self-Assessment and Help-Seeking

Accurate self-assessment enables appropriate help-seeking and strategy adjustment, both critical components of self-regulated learning [11]. Continuing the focus on effective planning, students often struggle with calibration between confidence and competence, with poor performers particularly prone to overestimating their abilities [42]. Simulated learners should express uncertainty appropriately, making statements like "I'm not sure if this is the right approach, I should check by trying a different method" to model self-appraisal. At this stage, these simulated learners should model metacognitive confidence judgments that align with actual performance, demonstrating how to gauge one's own understanding accurately [43]. Simulated learners should demonstrate productive help-seeking behaviors when stuck, such as consulting resources, reviewing worked examples, asking specific questions rather than general pleas for help, and identifying precisely where understanding breaks down [5]. The system should demonstrate simulated learners adjusting confidence based on solution verification, recognizing when initial certainty was misplaced. Designers should include both overconfident and underconfident simulated learners to facilitate discussions about calibration issues, helping students recognize both types of miscalibration in themselves.

3.7 Principle 7: Dialogic Interaction and Reciprocal Teaching

Reciprocal teaching, where students alternate between teaching and learning roles, enhances both comprehension and metacognition by requiring learners to externalize their thinking and respond to questions [44]. Explaining to others deepens understanding because it requires learners to organize knowledge coherently, identify gaps in understanding, and repair misconceptions [45]. The system should enable students to teach simulated learners, requiring them to explain procedures and concepts that prompt metacognitive reflection on their own understanding. Simulated learners should be programmed to ask clarifying questions about student explanations, such as "Why did you choose that method?" or "Can you explain what that variable represents?" to promote deeper thinking. The environment should include peer-like dialogue in which students and simulated learners collaboratively solve problems, negotiate approaches, and evaluate each other's suggestions. Designers should implement "teachable agents" that improve based on student instruction quality, providing motivation through visible progress that results from student teaching [46, 47]. The system should require students to provide feedback on simulated learner performance, reversing the typical power dynamic and positioning students as evaluators of mathematical reasoning.

3.8 Principle 8: Domain-Specific Metacognitive Strategies

Effective metacognitive instruction must be embedded within specific content domains rather than taught as generic, context-free skills, because metacognitive processes are highly dependent on domain knowledge and problem structure [48]. Algebra has unique metacognitive demands related to symbolic manipulation, equation structure, and the translation between verbal and symbolic representations [49]. Simulated learners should focus metacognitive prompts on algebra-specific challenges such as understanding the meaning of variables, maintaining equation balance through equivalent transformations, interpreting symbolic notation correctly, and recognizing structural patterns in expressions. These simulated learners should model strategies specific to algebraic problem types, including word-problem translation, solution approaches to systems of equations, and factoring decision-making. Simulated learners should include domain-relevant monitoring questions such as "Did I perform the same operation on both sides of the equation?" "Have I simplified this expression completely?" and "Does this factorization check when I expand it?" Simulated learners should demonstrate metacognitive awareness of common algebraic misconceptions, such as errors with the distributive property, sign errors in equation solving, and incorrect application of exponent rules, helping students anticipate and avoid these pitfalls.

3.9 Principle 9: Affective and Motivational Authenticity

Motivation, emotion, and self-efficacy significantly influence self-regulated learning, with these affective factors often determining whether students persist with challenging tasks or disengage [50–52]. Mathematical anxiety and negative self-beliefs hinder metacognitive engagement because anxious students tend to rush through problems without adequate planning or monitoring [14]. Simulated learners should express realistic emotional responses to both challenge and success, acknowledging difficulty without catastrophizing and celebrating progress appropriately. These simulated learners should model productive responses to confusion, frustration and setbacks, including perseverance, strategy adjustment, reframing challenges as learning opportunities, and appropriate help-seeking [53, 54]. The system should include simulated learners with varied confidence levels and anxiety responses to normalize diverse emotional experiences and show students that struggling is a common part of learning. Simulated learners should show how metacognitive strategies help manage mathematical anxiety, such as using planning to reduce overwhelming feelings and monitoring to build confidence through verified progress.

3.10 Principle 10: Formative Feedback with Metacognitive Reflection

Feedback is most effective when it prompts metacognitive reflection rather than simply providing correct answers, so learners can understand why their approaches succeeded or failed [55, 56]. Students need opportunities to compare their performance with standards and adjust strategies accordingly, treating feedback as information for self-regulation rather than as evaluation [57]. The system should provide feedback that targets both cognitive and metacognitive dimensions of performance, such as algebraic accuracy, planning adequacy, and monitoring effectiveness. Simulated learners should prompt students to reflect on discrepancies between their approach and the simulated learner's, asking "How was your strategy different from what the simulated learner did?" and "Which approach do you think was more efficient, and why?" The environment should include "What would you do differently next time?" prompts after problem completion to encourage forward-looking reflection that transfers to future problems and modifications to metacognitive conditional knowledge and skills [10]. The system should offer process-focused feedback emphasizing strategy effectiveness rather than just solution correctness, helping students understand that how they solve problems matters as much as whether they reach correct answers. The system should enable students to review simulated learner solutions and justify their agreement or disagreement with the approaches taken, thereby developing critical evaluation skills perhaps mediated and scaffolded by an embedded pedagogical agent.

3.11 Principle 11: Progressive Complexity and Challenge Adaptation

Learning environments should provide appropriately challenging tasks that require metacognitive engagement while remaining within students' zone of proximal development, balancing support and challenges to optimize learning [58]. Adaptive difficulty maintains engagement and prevents both boredom from tasks that are too easy and cognitive overload from tasks that are too difficult [27]. The system should implement adaptive algorithms that adjust simulated learner complexity based on student performance, providing more scaffolding when students struggle and reducing support as competence develops. Designers should structure sequences that begin with highly scaffolded examples in which simulated learners demonstrate complete solutions with extensive metacognitive commentary and progress to independent problem-solving, where students work with minimal support. The environment should vary problem complexity to require increasing metacognitive sophistication, starting with single-step problems that require minimal planning and advancing to multi-step problems that demand careful goal decomposition and strategic planning. The system should provide "hints" through simulated learner thinking when students struggle repeatedly, offering just-in-time support without eliminating productive struggle. Designers should balance worked examples from simulated learners with opportunities for independent practice, following research showing that interleaving examples and practice problems enhances learning.

3.12 Principle 12: Context-Rich and Meaningful Problem Situations

Contextualized problems enhance motivation and support transfer of learning by helping students see connections between abstract algebraic procedures and meaningful applications [59]. Authentic applications help students understand when and why to apply algebraic methods, addressing the common student complaint that algebra lacks real-world relevance [60]. Simulated learners should work on realistic, contextually relevant algebra problems that align with adolescent interests (e.g., social media), such as financial planning, sports statistics, technology applications, and social situations. They should model translation between verbal problem descriptions and algebraic representations, demonstrating how to extract mathematical structure from contextual information. Furthermore, the simulated learners should demonstrate metacognitive awareness of context, asking questions like "What do these variables represent in the real situation?" and "Does my answer make sense given what the problem is about?" The system should include problems where simulated learners discuss whether solutions make practical sense in context, such as recognizing that negative time values are meaningless or that fractional people are impossible. Simulated learners should demonstrate connections between algebraic procedures and meaningful real-world applications, helping students appreciate algebra's power and utility.

3.13 Principle 13: Bidirectional Learner-Simulated Learner Adaptation Using Multimodal Data

The design principle of adaptive bidirectional responsiveness through multimodal trace data (e.g., log-files, eye movements, facial expressions of emotions, physiology signatures) exchange represents a sophisticated approach to fostering metacognition and self-regulated learning in algebra problem-solving environments based on recent work on human-AI hybrid learning [26, 61–63]. By creating genuine, interactive partnerships in which both human and simulated learners analyze each other's behavioral patterns (including the learners' actual multimodal data and simulated learners synthetic multimodal data) and adapt their subsequent actions accordingly, this principle moves beyond static instructional content toward dynamic learning experiences that mirror the responsive, contingent nature of effective human tutoring and collaboration. For example, simulated learners should adapt their difficulty level and support needs based on student competence indicators in trace data [27]. As students demonstrate increasing metacognitive sophistication through their behavioral patterns, as evidenced by consistent planning, systematic monitoring, effective error correction, and strategic flexibility, simulated learners should gradually require less support and guidance, presenting students with more complex diagnostic challenges that maintain appropriate cognitive demand. Simulated learners should provide adaptive feedback that responds to specific patterns in student trace data rather than generic encouragement. When a student's behavioral data shows improvement in metacognitive skills, such as increased planning time, more frequent checking of problem-solving actions, or more strategic help-seeking the simulated learner should explicitly acknowledge these process improvements with comments like "I noticed you spent more time analyzing this problem before solving it and I think that's why your solution was more efficient" [56].

The pedagogical power of bidirectional learner-simulated learner adaptation using multimodal data emerges from multiple mechanisms: students develop metacognitive awareness by diagnosing simulated learner needs from trace data; (2) simulated learners provide timely modeling and scaffolding based on detected student difficulties; (3) collaborative problem-solving distributes cognitive load while maintaining high engagement; and (4) bidirectional feedback loops create accountability for both teaching, learning, and problem solving. When implemented with careful attention to technical sophistication, pedagogical integration, ethical data practices, and empirical validation, bidirectional adaptive systems can transform simulated learners into responsive "intelligent" learning partners that actively support students' development of metacognition and SRL. The ultimate goal is to create ILEs where metacognitive competencies emerge not through explicit instruction alone, but through authentic, adaptive interactions that make invisible regulatory processes visible, analyzable, and ultimately internalizable by students who are learning to become effective monitors and regulators of their own mathematical learning and problem-solving.

4 Conclusion and Future Directions

The design principles articulated in this paper represent an initial framework for developing simulated learners that externalize metacognition and self-regulation to support high school students' development of algebraic word-problem solving competencies. By integrating research from metacognition and self-regulated learning, these principles demonstrate how simulated learners can surpass traditional worked examples or static pedagogical agents to become dynamic, responsive learning partners that make invisible cognitive and metacognitive processes visible and teachable. The multimodal trace data integration principle enables unprecedented insights into both human and simulated learner problem-solving processes, creating opportunities for diagnostic analysis, comparative reflection, and targeted intervention based on fine-grained behavioral patterns.

The bidirectional adaptive responsiveness principle establishes simulated learners as genuine collaborative partners who model effective self-regulatory strategies and adapt their behavior in response to detected student needs, creating authentic interactive dynamics in which teaching and learning roles fluidly interchange. Together, these principles recognize that metacognitive development emerges not solely through explicit instruction but through situated practice within supportive environments. Specifically, they scaffold awareness, provide models of expert thinking, create opportunities for explanation and teaching, and gradually transfer regulatory responsibility from external agents to internalized self-direction. When implemented with careful attention to pedagogical integration, technical sophistication, ethical data practices, transparency, and learner agency, simulated learners designed according to these principles have potential to transform algebra learning environments from content delivery systems into metacognitively rich spaces that prepare students not just to solve equations but to become strategic, self-aware, adaptive mathematical thinkers capable of monitoring their own understanding and regulating their learning processes across diverse contexts.

Several promising directions for future research and development emerge from this set of principles that warrant systematic investigation. First, empirical validation studies are essential to establish which specific design features contribute most significantly to metacognitive development and algebraic competence, examining questions such as whether certain combinations of simulated learner behaviors are particularly powerful, how individual differences in prior knowledge and mathematical anxiety moderate effectiveness, and what temporal dynamics of scaffolding and fading optimize the transition from supported to independent problem-solving. Second, comparative studies should investigate the relative benefits of different simulated learner interaction modalities, such as whether teaching simulated learners produces stronger metacognitive gains than observing simulated learner models or collaborating as co-solvers, and whether multimodal trace data visualizations enhance or potentially overwhelm students' capacity for self-reflection. Third, longitudinal research is needed to assess whether metacognitive competencies developed through interactions with simulated learners transfer to non-scaffolded contexts, persist over extended time periods, and generalize across mathematical domains beyond algebra. This would address the critical question of whether these environments produce genuine self-regulatory expertise or scaffold-dependent performance. Lastly, research should explore the optimal balance between human teacher guidance and simulated learner support, investigating how classroom pedagogical practices can most effectively leverage insights from student-simulated learner interactions and how teachers' metacognitive modeling complements or potentially conflicts with simulated learner demonstrations.

From a technological development perspective, several advances would enhance the sophistication and pedagogical power of simulated learner systems. For example, natural language processing capabilities that enable simulated learners to engage in more nuanced, open-ended dialogue about problem-solving strategies would support deeper metacognitive discourse than current menu-based or constrained interaction systems typically allow. The integration of advanced affective computing techniques, including facial expression analysis, voice stress detection, and sophisticated physiological measurement, would enable more sensitive detection of emotional states like frustration, confusion, and flow, allowing simulated learners to respond with appropriate emotional support and motivational scaffolding. Machine learning approaches that enable simulated learners to develop increasingly sophisticated student models over extended interactions could support truly personalized adaptation where simulated behaviors adjust not just to immediate trace data patterns but to individual students' evolving metacognitive profiles, preferred learning strategies, and developing competencies. Developing multi-agent systems in which students interact simultaneously with multiple simulated learners exhibiting diverse problem-solving approaches could create richer opportunities to compare strategies, mediate between alternative methods, and develop flexible, adaptive expertise. Furthermore, exploration of immersive technologies such as virtual and augmented reality could enable embodied simulated learners whose gestures, gaze patterns, and spatial positioning provide additional channels for metacognitive modeling. Also, as generative artificial intelligence (GenAI) capabilities continue to advance, simulated learners could be designed to engage in increasingly sophisticated mathematical discourse and "metacognitive talk", ask probing questions that genuinely challenge student thinking, and collaboratively explore novel problems in which neither student nor simulated learner have predetermined solution paths. These technological advances, combined with continued theoretical refinement grounded in foundational AI, and cognitive and leaning sciences research, position simulated learners as powerful tools for democratizing access to the kinds of metacognitively rich learning experiences that effective human teaching and tutoring provide, ultimately supporting all students in developing the self-regulatory competencies essential for mathematical success and lifelong learning.

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