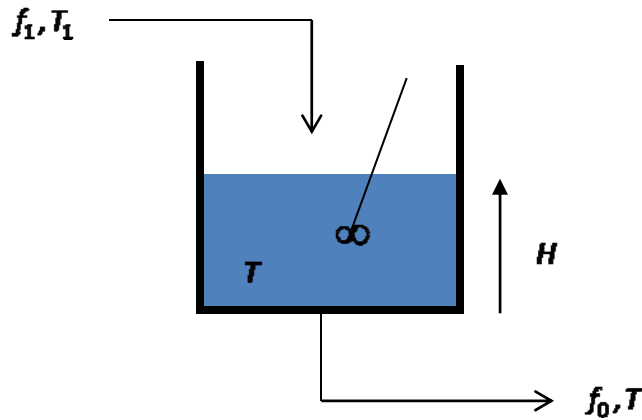


A single tank with cross-sectional area A ft² receives an inflow of $f_1(t)$ in ft³/min at a temperature of $T_1(t)$, measured in deg F. Outflow $f_0(t)$ is in ft³/min and at temperature $T(t)$, in deg F. The liquid level and temperature in the tank at time t is $H(t)$ and $T(t)$, respectively.



The tank is modeled by the following differential and algebraic equations:

$$A \frac{dH}{dt} + f_0 = f_1$$

$$f_0 = \alpha H^{1/2}$$

$$cf_1 T_1 - cf_0 T = c \frac{d}{dt}(AHT)$$

The last equation reflects a conservation of energy, i.e.

Rate of energy in – rate of energy leaving = rate of accumulation of energy

where c is the specific heat of the liquid measured in Btu / degF per ft³.

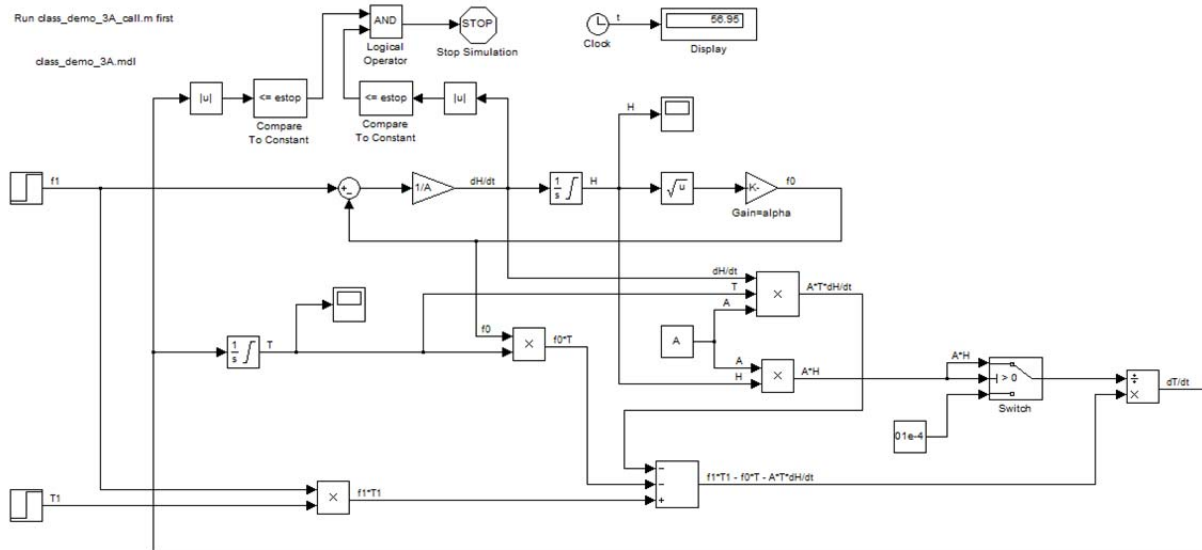
$$\frac{\text{Btu}}{\text{ft}^3 - \text{degF}} \cdot \frac{\text{ft}^3}{\text{min}} \cdot \text{degF} - \frac{\text{Btu}}{\text{ft}^3 - \text{degF}} \cdot \frac{\text{ft}^3}{\text{min}} \cdot \text{degF} = \frac{\text{Btu}}{\text{ft}^3 - \text{degF}} \cdot \frac{1}{\text{min}} \cdot (\text{ft}^2 \cdot \text{ft} \cdot \text{degF})$$

$$\frac{\text{Btu}}{\text{min}} - \frac{\text{Btu}}{\text{min}} = \frac{\text{Btu}}{\text{min}}$$

The right hand side of the last equation is expanded to

$$cf_1 T_1 - cf_0 T = cA \frac{d}{dt}(HT) = cA \left(H \frac{dT}{dt} + T \frac{dH}{dt} \right)$$

After solving for $\frac{dT}{dt}$ in the last equation, the Simulink diagram is obtained as shown.



```
% class_demo_3A.m
% example of a tank with two inputs, flow and temperature

clc, close all, clear all

A=10; % tank area
F1=12; % amplitude of step flow in
alpha=4; % discharge flow constant
H_init=10;
T_init=70;
delta_T=50; % change in temp step flow in above T_init
step_size=0.025; % RK-4 integration step size
tfinal=500;
estop=0.01; % stop condition for |dH/dt| and |dT/dt|
sim('class_demo_3A')
t=t_H(:,1);
H=t_H(:,2);
T=t_T(:,2);
subplot(2,1,1)
plot(t,H)
ylabel('H (ft)')
title('H vs t')
subplot(2,1,2)
plot(t,T)
xlabel('t (min)')
ylabel('T (deg F)')
title('T vs t')
```