Assignment #7 Key

Consider the boolean CNF expression E = (a+b+c+d)(~a)(~b+d)(a+b+~d)
Here + is or and catenation of terms is and.

1. Recast E in 3-CNF form (that is, with each term being a disjunct of three items)

E = (a+b+e)(c+d+~e)(~a+~a+~a)(~b+d+d)(a+b+~d)

2. Present the table that represents a conversion of E's satisfiability to an instance of SubsetSum

	a	b	c	d	e	a+b+e	c+d+~e	~a+~a+~a	~b+d+d	a+b+~d
a	**1**					**1**				**1**
~a	**1**							**3 (or 1)**		
b		**1**				**1**				**1**
~b		**1**							**1**	
c			**1**				**1**			
~c			**1**							
d				**1**			**1**		**2 (or 1)**	
~d				**1**						**1**
e					**1**	**1**				
~e					**1**		**1**			
C1						**1**			**1**	
C1'						**1**			**1**	
C2							**1**			
C2'							**1**			
C3								**1**		
C3'								**1**		
C4									**1**	
C4'									**1**	
C5										**1**
C5'										**1**
	1	**1**	**1**	**1**	**1**	**3**	**3**	**3**	**3**	**3**

3. Explicitly write down the numbers that comprise this instance of SubsetSum

1 0 0 0 0 1 0 0 0 1
1 0 0 0 0 0 0 3 0 0
0 1 0 0 0 1 0 0 0 1
0 1 0 0 0 0 0 0 1 0
0 0 1 0 0 0 1 0 0 0
0 0 1 0 0 0 0 0 0 0
0 0 0 1 0 0 1 0 2 0
0 0 0 1 0 0 0 0 0 1
0 0 0 0 1 1 0 0 0 0
0 0 0 0 1 0 1 0 0 0
0 0 0 0 0 1 0 0 0 0
0 0 0 0 0 1 0 0 0 0
0 0 0 0 0 0 1 0 0 0
0 0 0 0 0 0 1 0 0 0
0 0 0 0 0 0 0 1 0 0
0 0 0 0 0 0 0 1 0 0
0 0 0 0 0 0 0 0 1 0
0 0 0 0 0 0 0 0 1 0
0 0 0 0 0 0 0 0 1 0
0 0 0 0 0 0 0 0 0 1
0 0 0 0 0 0 0 0 0 1

4. Show a solution to this SubsetSum instance that encodes a solution to E's satisfiability

~a, b, c, d, e

1 0 0 0 0 0 0 3 0 0
0 1 0 0 0 1 0 0 0 1
0 0 1 0 0 0 1 0 0 0
0 0 0 1 0 0 1 0 2 0
0 0 0 0 1 1 0 0 0 0
0 0 0 0 0 1 0 0 0 0
0 0 0 0 0 0 1 0 0 0
0 0 0 0 0 0 0 0 1 0
0 0 0 0 0 0 0 0 0 1
0 0 0 0 0 0 0 0 0 1
= 1 1 1 1 1 3 3 3 3 3

5. Recast the SubsetSum instance you have as an instance of Partition

Add two numbers to set from 3. These are:

3 3 3 3 3 7 7 7 7 7 2*Sum - G
3 3 3 3 3 8 8 8 8 8 Sum + G

6. Show an explicit solution to this instance of Partition -- that's easy given (3)

P1

3 3 3 3 3 7 7 7 7 7
1 0 0 0 0 0 0 3 0 0
0 1 0 0 0 1 0 0 0 1
0 0 1 0 0 0 1 0 0 0
0 0 0 1 0 0 1 0 2 0
0 0 0 0 1 1 0 0 0 0
0 0 0 0 0 1 0 0 0 0
0 0 0 0 0 0 1 0 0 0
0 0 0 0 0 0 0 0 1 0
0 0 0 0 0 0 0 0 0 1
0 0 0 0 0 0 0 0 0 1
= 4 4 4 4 5 1 1 1 1 0

P2

3 3 3 3 3 8 8 8 8 8
0 1 0 0 0 0 0 0 1 0
0 0 1 0 0 0 0 0 0 0
0 0 0 1 0 0 0 0 0 1
0 0 0 0 1 0 1 0 0 0
0 0 0 0 0 1 0 0 0 0
0 0 0 0 0 0 1 0 0 0
0 0 0 0 0 0 0 1 0 0
0 0 0 0 0 0 0 1 0 0
0 0 0 0 0 0 0 0 1 0
= 4 4 4 4 5 1 1 1 1 0

1 0 0 0 0 1 0 0 0 1

7. Recast the 3-CNF form of E as an instance of k-Vertex Covering and present a solution to the latter
E = (a+b+e)(c+d+~e)(~a+~a+~a)(~b+d+d)(a+b+~d)
Look at notes on the needed gadgets and connections
The k-Vertex cover goal is the number of variables + 2*number of cluases = 5 + 10 = 15.

8. Recast the 3-CNF form of E as an instance of the k-Coloring problem and present a solution to the latter
E = (a+b+e)(c+d+~e)(~a+~a+~a)(~b+d+d)(a+b+~d)
Look at notes on the needed gadgets and connections. The k=3 here.