**Assignment #7 Key**

Consider the boolean CNF expression E = (a+b+c+d)(~a)(~b+d)(a+b+~d)

Here + is or and catenation of terms is and.

1. Recast E in 3-CNF form (that is, with each term being a disjunct of three items)

**E = (a+b+e)(c+d+~e)(~a+~a+~a)(~b+d+d)(a+b+~d)**

2. Present the table that represents a conversion of E's satisfiability to an instance of SubsetSum

|  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | a | b | c | d | e | a+b+e | c+d+~e | ~a+~a+~a | ~b+d+d | a+b+~d |
| **a** | **1** |  |  |  |  | **1** |  |  |  | **1** |
| **~a** | **1** |  |  |  |  |  |  | **3 (or 1)** |  |  |
| **b** |  | **1** |  |  |  | **1** |  |  |  | **1** |
| **~b** |  | **1** |  |  |  |  |  |  | **1** |  |
| **c** |  |  | **1** |  |  |  | **1** |  |  |  |
| **~c** |  |  | **1** |  |  |  |  |  |  |  |
| **d** |  |  |  | **1** |  |  | **1** |  | **2 (or 1)** |  |
| **~d** |  |  |  | **1** |  |  |  |  |  | **1** |
| **e** |  |  |  |  | **1** | **1** |  |  |  |  |
| **~e** |  |  |  |  | **1** |  | **1** |  |  |  |
| **C1** |  |  |  |  |  | **1** |  |  | **1** |  |
| **C1’** |  |  |  |  |  | **1** |  |  | **1** |  |
| **C2** |  |  |  |  |  |  | **1** |  |  |  |
| **C2’** |  |  |  |  |  |  | **1** |  |  |  |
| **C3** |  |  |  |  |  |  |  | **1** |  |  |
| **C3’** |  |  |  |  |  |  |  | **1** |  |  |
| **C4** |  |  |  |  |  |  |  |  | **1** |  |
| **C4’** |  |  |  |  |  |  |  |  | **1** |  |
| **C5** |  |  |  |  |  |  |  |  |  | **1** |
| **C5’** |  |  |  |  |  |  |  |  |  | **1** |
|  | **1** | **1** | **1** | **1** | **1** | **3** | **3** | **3** | **3** | **3** |

3. Explicitly write down the numbers that comprise this instance of SubsetSum

**1 0 0 0 0 1 0 0 0 1**

**1 0 0 0 0 0 0 3 0 0**

**0 1 0 0 0 1 0 0 0 1**

**0 1 0 0 0 0 0 0 1 0**

**0 0 1 0 0 0 1 0 0 0**

**0 0 1 0 0 0 0 0 0 0**

**0 0 0 1 0 0 1 0 2 0**

**0 0 0 1 0 0 0 0 0 1**

**0 0 0 0 1 1 0 0 0 0**

**0 0 0 0 1 0 1 0 0 0**

**0 0 0 0 0 1 0 0 0 0**

**0 0 0 0 0 1 0 0 0 0**

**0 0 0 0 0 0 1 0 0 0**

**0 0 0 0 0 0 1 0 0 0**

**0 0 0 0 0 0 0 1 0 0**

**0 0 0 0 0 0 0 1 0 0**

**0 0 0 0 0 0 0 0 1 0**

**0 0 0 0 0 0 0 0 1 0**

**0 0 0 0 0 0 0 0 1 0**

**0 0 0 0 0 0 0 0 0 1**

**0 0 0 0 0 0 0 0 0 1**

4. Show a solution to this SubsetSum instance that encodes a solution to E's satisfiability

**~a, b, c, d, e**

**1 0 0 0 0 0 0 3 0 0**

**0 1 0 0 0 1 0 0 0 1**

**0 0 1 0 0 0 1 0 0 0**

**0 0 0 1 0 0 1 0 2 0**

**0 0 0 0 1 1 0 0 0 0**

**0 0 0 0 0 1 0 0 0 0**

**0 0 0 0 0 0 1 0 0 0**

**0 0 0 0 0 0 0 0 1 0**

**0 0 0 0 0 0 0 0 0 1**

**0 0 0 0 0 0 0 0 0 1**

**= 1 1 1 1 1 3 3 3 3 3**

5. Recast the SubsetSum instance you have as an instance of Partition

**Add two numbers to set from 3. These are:**

**3 3 3 3 3 7 7 7 7 7 2\*Sum - G**

**3 3 3 3 3 8 8 8 8 8 Sum + G**

6. Show an explicit solution to this instance of Partition -- that's easy given (3)

**P1**

**3 3 3 3 3 7 7 7 7 7**

**1 0 0 0 0 0 0 3 0 0**

**0 1 0 0 0 1 0 0 0 1**

**0 0 1 0 0 0 1 0 0 0**

**0 0 0 1 0 0 1 0 2 0**

**0 0 0 0 1 1 0 0 0 0**

**0 0 0 0 0 1 0 0 0 0**

**0 0 0 0 0 0 1 0 0 0**

**0 0 0 0 0 0 0 0 1 0**

**0 0 0 0 0 0 0 0 0 1**

**0 0 0 0 0 0 0 0 0 1**

**= 4 4 4 4 5 1 1 1 1 0**

**P2**

**3 3 3 3 3 8 8 8 8 8**

**0 1 0 0 0 0 0 0 1 0**

**0 0 1 0 0 0 0 0 0 0**

**0 0 0 1 0 0 0 0 0 1**

**0 0 0 0 1 0 1 0 0 0**

**0 0 0 0 0 1 0 0 0 0**

**0 0 0 0 0 0 1 0 0 0**

**0 0 0 0 0 0 0 1 0 0**

**0 0 0 0 0 0 0 1 0 0**

**0 0 0 0 0 0 0 0 1 0**

**= 4 4 4 4 5 1 1 1 1 0**

**1 0 0 0 0 1 0 0 0 1**

7. Recast the 3-CNF form of E as an instance of k-Vertex Covering and present a solution to the latter

**E = (a+b+e)(c+d+~e)(~a+~a+~a)(~b+d+d)(a+b+~d)**

**Look at notes on the needed gadgets and connections**

**The k-Vertex cover goal is the number of variables + 2\*number of cluases = 5 + 10 = 15.**

8. Recast the 3-CNF form of E as an instance of the k-Coloring problem and present a solution to the latter

**E = (a+b+e)(c+d+~e)(~a+~a+~a)(~b+d+d)(a+b+~d)**

**Look at notes on the needed gadgets and connections. The k=3 here.**