Assignment #5 Key; Due February 20 at start of class

1. Consider the set of indices **NonConstant = NC = { f | |range(φ_f)| > 1 }**. Use Rice's Theorem to show that **NC** is not recursive (not decidable). Note that members of **NC** do not need to converge for all input, but they must converge on at least two input values that produce different output values. Hint: There are two properties that must be demonstrated.

   First, NC is non-trivial as I(x) = x is in NC and Z(x) = 0 is not.

   Second, NC is an I/O Property.
   To see this, let f and g be arbitrary indices of computable functions such that
   $\forall x\ \varphi_f(x) = \varphi_g(x)$.
   f is in NC iff |range(φ_f)| > 1. But g's range is exactly that of f and so,
   |range(φ_f)| > 1 iff |range(φ_g)| > 1. But then,
   f is in NC iff g is in NC

   Since NC is not trivial and is an I/O property then it is not recursive by Rice's Theorem.

2. Show that **K $\leq_m$ NonConstant**, where **K = { f | φ_f(f)↓ }**.

   Let f be an arbitrary index of some computable function.
   Then, f is in K iff $\varphi_f(f)\downarrow$
   Define $g_f(x) = \varphi_f(f) - \varphi_f(f) + x$
   For all x, $g_f(x) = x$ iff $\varphi_f(f)\downarrow$; and diverges otherwise.
   But then f is in K iff g_f is in NC.
   This shows that K $\leq_m$ NonConstant, as was desired.