**Assignment #5 Key; Due February 20 at start of class**

1. Consider the set of indices **NonConstant = NC = { f | |range(ϕf)| > 1 }**. Use Rice’s Theorem to show that **NC** is not recursive (not decidable). Note that members of **NC** do not need to converge for all input, but they must converge on at least two input values that produce different output values. Hint: There are two properties that must be demonstrated.

***First, NC is non-trivial as I(x) = x is in NC and Z(x) = 0 is not.***

***Second, NC is an I/O Property.   
To see this, let f and g be arbitrary indices of computable functions such that  
∀x ϕf(x) = ϕg(x).***

***f is in NC iff |range(ϕf)| > 1. But g’s range is exactly that of f and so,***

***|range(ϕf)| > 1 iff |range(ϕg)| > 1. But then,***

***f is in NC iff g is in NC***

***Since NC is not trivial and is an I/O property then it is not recursive by Rice’s Theorem.***

1. Show that **K** ≤**m NonConstant**, where **K = { f |** ϕ**f(f)**↓ **}**.

***Let f be an arbitrary index of some computable function.   
Then, f is in K iff ϕf(f)↓***

***Define gf (x) = ϕf(f) - ϕf(f) + x***

***For all x, gf (x) = x iff ϕf(f)↓; and diverges otherwise.***

***But then f is in K iff gf  is in NC.***

***This shows that K ≤m NonConstant, as was desired.***