Assignment #3 Key; Due February 6 at start of class

Show that prfs are closed under mutual induction. Mutual induction means that each induction step after calculating the base is computed using the previous value of the other function. The formal hypothesis is:
Assume g1, g2, g3, h1, h2 and h3 are already known to be prf, then so are f1, f2 and f3, where
f1(x,0) = g1(x); f1(x,y+1) = h1(f2(x,y),f3(x,y));
f2(x,0) = g2(x); f2(x,y+1) = h2(f3(x,y),f1(x,y))
f3(x,0) = g3(x); f3(x,y+1) = h3(f1(x,y),f2(x,y))

Prove by construction:

Let **K** be the following primitive recursive function, defined by induction on the primitive recursive functions, **g1**, **g2**, **g3**, **h1**, **h2**, **h3** and the extended pairing function that maps triples into single numbers.

K(x,0) = B(x)
// this is just $\langle f1(x,0), f2(x,0), f3(x,0) \rangle$
$B(x) = \langle g1(x), g2(x), g3(x) \rangle$
K(x, y+1) = J(x, y, K(x, y))
// this is $\langle f1(x,y+1), f2(x,y+1), f3(x,y+1) \rangle$
$J(x, y, z) = \langle h1(\langle z\rangle_2,\langle z\rangle_3), h2(\langle z\rangle_3,\langle z\rangle_1), h3(\langle z\rangle_1,\langle z\rangle_2) \rangle$
This shows **K** is prf. **f1**, **f2** and **f3** are then defined from **K** as follows:
$f1(x,y) = \langle K(x,y)\rangle_1$ // extract first value from triple encoded in **K(x,y)**
$f2(x,y) = \langle K(x,y)\rangle_2$ // extract second value from triple encoded in **K(x,y)**
$f3(x,y) = \langle K(x,y)\rangle_3$ // extract third value from triple encoded in **K(x,y)**
This shows **f1**, **f2** and **f3** are also prf, as was desired.