Assignment#2 Key; Due January 28 at start of class

Let set **A** be non-empty recursive, **B** be re non-recursive and **C** be non-re. Using the terminology **(REC) recursive**, **(RE) non-recursive recursively enumerable**, **(NR) non-re**, categorize each set below, saying whether or not the set can be of the given category and justifying each answer. You may assume, for any set **S**, the existence of comparably hard sets
$\mathbf{S_E}$ **= {2x|x∈S}** and $\mathbf{S_D}$ **= {2x+1|x∈S}**. The following is a sample of the kind of answer I require:

Sample.) **A ∩ C = { x | x ∈ A and x ∈ C }**

REC: Yes. If A = {0} then A ∩ C = ∅ ot {0}, each of which is in REC.

RE: Yes. Let A = $\aleph_E$ **= { 2x | x ∈ ℵ }; let C =** TOT_D **∪** $HALT_E$ **then A ∩ C =** $HALT_E$ **which is in RE**

NR: Yes. If A = ℵ then A ∩ C = C, which is in NR.

a.) **B – A = { x | x ∈ B and x ∉ A } // Set difference**

REC: Yes. Let A = ℵ, B = HALT, then B – A = ∅, which is in REC

RE: Yes. Let A = {0}, B = HALT, then B – A = HALT – {0}, which is in RE

NR: This is not possible. To see this, consider that B – A = B ∩ ~A. Since A is in REC, then so is ~A. A semi-decision procedure for B – A cab be constructed by first seeing the chosen number, call it x, is in ~A. If it is not then answer "NO." If it is then run the semi-decision procedure for B. If it ever answers "YES," produce the answer "YES." Formally, if χ_A **decides A and** g_B **semi-decides B, the** $g_{B-A}(x) = (1-\chi_A(x)) * g_B(x)$ **semi-decide B – A.**

b.) **A* B = { x * y | x ∈ A and y ∈ B } // Multiplication**

REC: Let A = {0}, B = HALT, then A * B = {0} , which is in REC

RE: Let A = {1}, B = HALT, then A * B = HALT , which is in RE

NR: This is not possible. To see this, we need just show that we can semi-decide A * B. By definition x ∈ A * B iff ∃ a∈A, b∈B such that x = a*b. But, the rules of multiplication of natural numbers then implies we can limit our search to values of a,b such 0≤a,b≤x, so we have bounds on each value. To take advantage of this, let's get rid of 0 first. If x = 0, then x is in A * B, if 0 is in A (quick check) or 0 is in B (semi-decidable). Thus, we can semi-decide this case. For all other cases, just search for each a, 1≤a≤x such that a divides x. Include each such x/a in a list called Check. Now run the enumerating function for B to see if any of these items show up. If any does, answer "yes." This provides a semi-decision procedure.

c.) **A ∪ C = { x | x ∈ A or x ∈ C } // Set union**

REC: Let A = ℵ, C = ~HALT, then A ∪ C = ℵ, which is in REC

RE: Let A = $\aleph_D$**, B =** $HALT_E \cup \sim HALT_D$**, then A ∪ C =** $HALT_E \cup \aleph_D$**, which is in RE**

NR: Let A = {0}, C = ~HALT, then A ∪ C = ~HALT∪{0}, which is in NR