

Assignment#2; Due January 28 at start of class

Let set **A** be non-empty recursive, **B** be re non-recursive and **C** be non-re. Using the terminology **(REC) recursive**, **(RE) non-recursive recursively enumerable**, **(NR) non-re**, categorize each set below, saying whether or not the set can be of the given category and justifying each answer. You may assume, for any set **S**, the existence of comparably hard sets $S_E = \{2x|x \in S\}$ and $S_D = \{2x+1|x \in S\}$. The following is a sample of the kind of answer I require:

Sample.) $A \cap C = \{x \mid x \in A \text{ and } x \in C\}$

REC: Yes. If $A = \{0\}$ then $A \cap C = \emptyset$ or $\{0\}$, each of which is in REC.

RE: Yes. Let $A = \aleph_E = \{2x \mid x \in \aleph\}$; let $C = \text{TOT}_D \cup \text{HALT}_E$ then $A \cap C = \text{HALT}_E$ which is in RE

NR: Yes. If $A = \aleph$ then $A \cap C = C$, which is in NR.

- a.) $B - A = \{x \mid x \in B \text{ and } x \notin A\}$ // Set difference
- b.) $A * B = \{x * y \mid x \in A \text{ and } y \in B\}$ // Multiplication
- c.) $A \cup C = \{x \mid x \in A \text{ or } x \in C\}$ // Set union

Be careful: Some may not be possible. If so, you must justify why this is so.

Note:

$\text{TOT} = \{x \mid \forall \varphi_x(y) \downarrow\}$. These are the indices of the set of algorithms.

$\text{HALT} = \{\langle x, y \rangle \mid \varphi_x(y) \downarrow\}$. This is the set of pairs of procedures and input for which the given procedure halts.

The set S_E for any set S is defined as $\{2x \mid x \in S\}$
 $S_D = \{2x+1 \mid x \in S\}$.