Assignment#1 Key; Due January 19 at start of class
Review of Formal Languages

Consider some language **L**. For each of the following cases, write in one of **(i)** through **(vi)**, to indicate what you can say conclusively about **L**'s complexity, where

(i) **L** is definitely regular
(ii) **L** is context-free, possibly not regular, but then again it might be regular
(iii) **L** is context-free, and definitely not regular
(iv) **L** might not even be context-free, but then again it might even be regular
(v) **L** is definitely not regular, and it may or may not be context-free
(vi) **L** definitely is not even context-free

Follow each answer with example languages **A** (and **B**, where appropriate) to back up the complexity claims inherent in your answer; and/or state some known closure property that reflects a bound on the complexity of **L**.

Example.) $L = A \cup B$, where **A** and **B** are both context free, and definitely not regular
L can be characterized by **Property (ii)**, above.
L is context-free, since the class of context-free languages is closed under union.
L can be regular. For example,
$A = \{ a^n b^m \mid m \geq n \}$, $B = \{ a^n b^m \mid m \leq n \}$,
$L = A \cup B = \{ a^n b^m \mid n, m \geq 0 \}$, which is regular since it can be represented by the regular expression **a*b***.
But **L** is in general not guaranteed to be regular, e.g., if we just make **A** and **B** the same context-free, non-regular set, then $L = A \cup A = A$, which is not regular.

a.) $L = A - B$, where **A** is context-free, non-regular and **B** is regular
(ii)
Regular: $A = a^n b^n$, $B = a^*b^*$, $L = \varnothing$
Context-Free: $A = a^n b^n$, $B = \varnothing$, $L = a^n b^n$
Non-Context-Free (**NO**): Cannot be so as $A - B = A \cap \sim B$. $\sim B$ is regular since regular are closed under complement and context free are closed under intersection with regular.

b.) $L = B - A$, where **A** is context-free, non-regular and **B** is regular
(iv)
Regular: $A = a^n b^n$, $B = \varnothing$, $L = \varnothing$
Context-Free: $A = a^n b^n$, $B = a^*b^*$, $L = \{ a^n b^m \mid n \neq m \}$
Non-Context-Free: $A = \{ a^s b^t c^u \mid s \neq t \text{ or } t \neq u \}$, $B = a^*b^*c^*$, $L = a^n b^n c^n$

c.) $A = L - B$, where **A** is context-free, non-regular and **B** is regular
(v)
Regular (**NO**): Cannot be so as $L - B = A \cap \sim B$. $\sim B$ is regular since regular are closed under complement and so L –B is regular since regular are closed under intersection.
Context-Free: $A = a^n b^n$, $L = a^n b^n$, $B = \varnothing$
Non-Context-Free: $A = a^n b^n$, $L = a^n b^n d^{n+1}.\cup a^n b^n$, $L = a^n b^n$, $B = a^*b^*d+$

d.) $L \subset A$, where **A** is context-free, but not regular
(iv)
Regular: $A = a^n b^n c^n$, $L = \varnothing$
Context-Free: $A = \{a^n b^n c^m, \mid m \neq n\}$, $L = a^{n+1} b^{n+1}$
Non-Context-Free: $A = a^n b^n c^n$, $L = a^{2n} b^{2n} c^{2n}$