**Assignment#1 Key; Due January 19 at start of class  
Review of Formal Languages**

Consider some language **L**. For each of the following cases, write in one of **(i)** through **(vi)**, to indicate what you can say conclusively about **L**’s complexity, where

**(i)** **L** is definitely regular

**(ii)** **L** is context-free, possibly not regular, but then again it might be regular

**(iii)** **L** is context-free, and definitely not regular

**(iv)** **L** might not even be context-free, but then again it might even be regular

**(v)** **L** is definitely not regular, and it may or may not be context-free

**(vi) L** definitely is not even context-free

Follow each answer with example languages **A** (and **B**, where appropriate) to back up the complexity claims inherent in your answer; and/or state some known closure property that reflects a bound on the complexity of **L**.

**Example.) L = A ∪ B**, where **A** and **B** are both context free, and definitely not regular

**L** can be characterized by **Property (ii)**, above.

**L** is context-free, since the class of context-free languages is closed under union.

**L** can be regular. For example,

**A = { an bm | m ≥ n }**, **B = { an bm | m ≤ n }**,

**L = A ∪ B = { an bm | n, m ≥ 0 }**, which is regular since it can be represented by the regular expression **a\*b\***.

But **L** is in general not guaranteed to be regular, e.g., if we just make **A** and **B** the same context-free, non-regular set, then **L = A ∪ A = A** , which is not regular.

**a.) L = A  B**, where **A** is context-free, non-regular and **B** is regular

**(ii)**

Regular: **A = anbn, B = a\*b\*, L = ∅**

Context-Free: **A = anbn, B = ∅, L = anbn**

Non-Context-Free (**NO**): Cannot be so as **A  B = A ∩ ~B**. **~B** is regular since regular are closed under complement and context free are closed under intersection with regular.

**b.) L = B  A**, where **A** is context-free, non-regular and **B** is regular

**(iv)**

Regular: **A = anbn, B = ∅, L = ∅**

Context-Free: **A = anbn, B = a\*b\*, L = { anbm | n≠m }**

Non-Context-Free: **A = { asbtcu | s≠t or t≠u }, B = a\*b\*c\*, L = anbncn**

**c.) A = L  B**, where **A** is context-free, non-regular and **B** is regular

**(v)**

Regular (**NO**): Cannot be so as **L  B = A ∩ ~B**. **~B** is regular since regular are closed under complement and so L –B is regular since regular are closed under intersection.

Context-Free: **A = anbn, L = anbn, B = ∅**

Non-Context-Free: **A = anbn, L = anbndn+1.∪ anbn, L = anbn, B = a\*b\*d+**

**d.) L ⊂ A**, where **A** is context-free, but not regular

**(iv)**

Regular: **A = anbncn, L = ∅**

Context-Free: **A = {anbncm, | m≠n}, L = an+1bn+1**

Non-Context-Free: **A = anbncn, L = a2nb2nc2n**