Complexity analysis of Evolution of Dual Preference Orderings in Games of International Conflict

Ramya Pradhan
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Overview

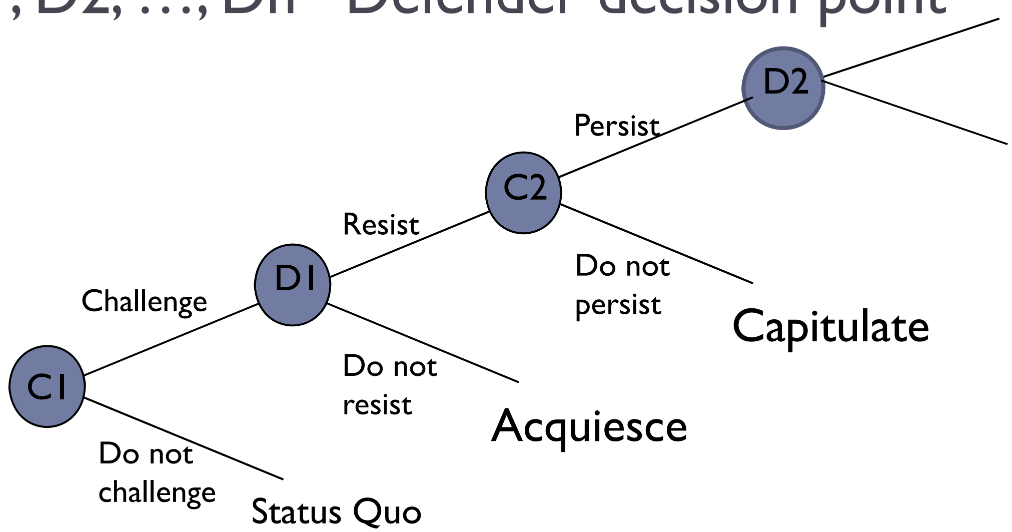
- Introduction
- Motivation
- Modeling
- Complexity Analysis
- Implication of Complexity Analysis
- References

Introduction

- International Conflict
  - General disagreement
  - Conflict – an agent of change
  - Models of conflict resolution
    - Conflict Game, Deterrence Game
- Conflict Game
  - Overarching concept of Deterrence Game
  - Two-player sequential game

Deterrence Game

- Model of
  - persuasion and ideology exchange
  - initiation and avoidance of war in international relations
- Decision Tree
  - C1, C2, …, Cn – Challenger decision points
  - D1, D2, …, Dn– Defender decision point



Deterrence Game

- Outcomes: leaf nodes
  - For example,
    - Status Quo (S) – no change
    - Acquiesce (A) – defender gives in
    - Capitulate (C) – challenger gives in

- Payoff Matrix: payoff received for decision made

Defender moves

Challenger moves

	Defend	Do not defend
Challenge	--	A (2,0)
Do not challenge	C (0,2)	S (1,1)

0 – player lost the game/replaced

1 – player unchanged

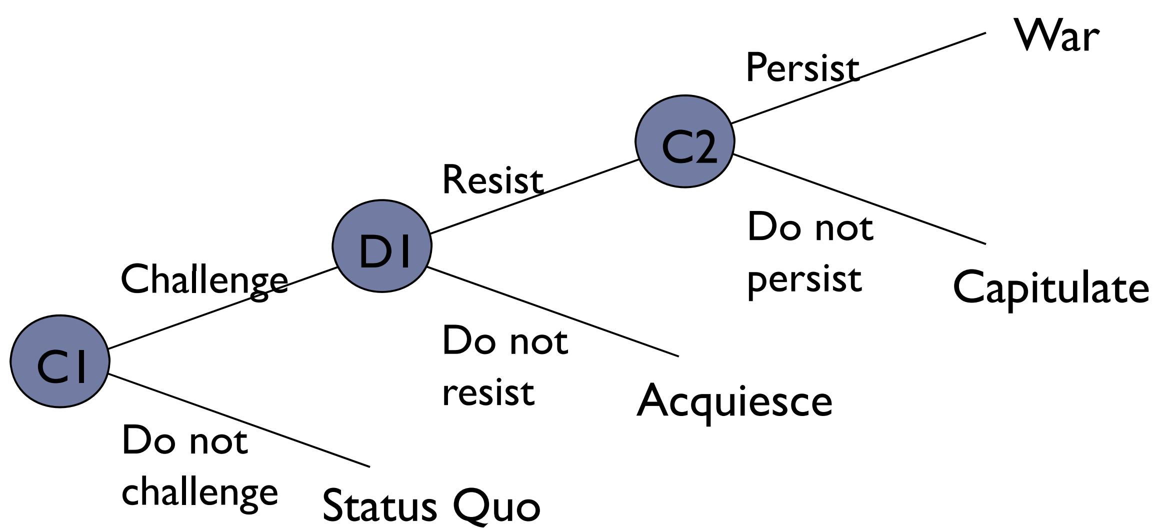
2 – player won the game

-- - we do not know as this is an game is played indefinitely

Source: [1]

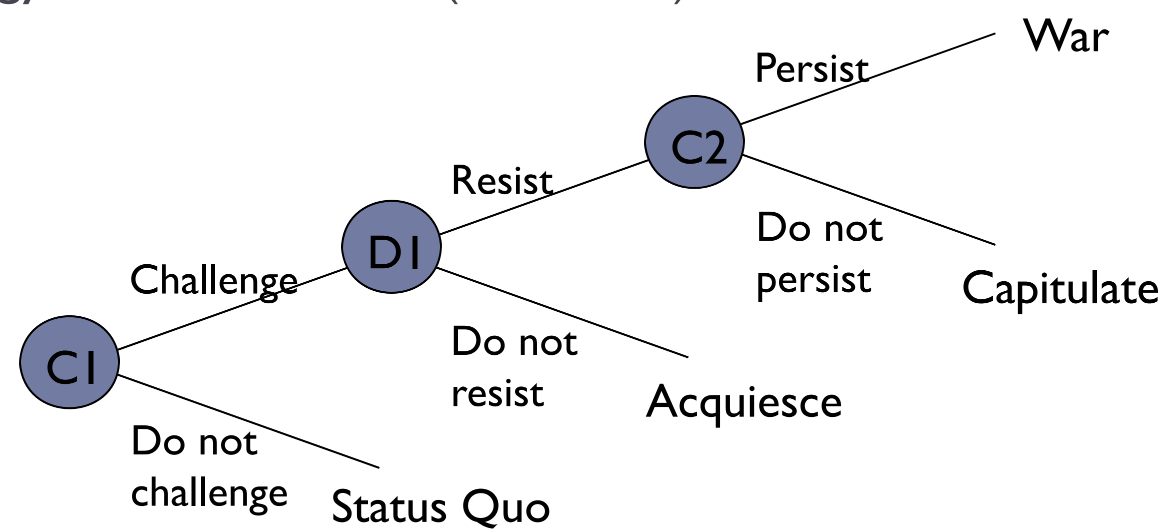
Deterrence Game

- How to play the deterrence game?
  - Each player has a strategy
  - Complete or incomplete information
  - With complete information, decision making starts at leaf node (our research)
  - Decision tree for our research



Deterrence Game

- Illustration:
  - P1's strategy: A > C > S > W (Challenger)
  - P2's strategy: C > S > A > W (Defender)



  - At C2: Capitulate
  - At D1: Capitulate
  - At C1: Capitulate
  - Outcome: Capitulate, Payoffs: Challenger – 0, Defender – 2

Deterrence Game

- 24 possible strategies

Code Letter	Payoff Vector	Preference Ordering	Code Letter	Payoff Vector	Preference Ordering
@	$(1,2,3,4)$	$WAR > CAP > ACQ > SQ$	l	$(3,1,2,4)$	$WAR > SQ > CAP > ACQ$
a	$(1,2,4,3)$	$CAP > WAR > ACQ > SQ$	m	$(3,1,4,2)$	$CAP > SQ > WAR > ACQ$
b	$(1,3,2,4)$	$WAR > ACQ > CAP > SQ$	n	$(3,2,1,4)$	$WAR > SQ > ACQ > CAP$
c	$(1,3,4,2)$	$CAP > ACQ > WAR > SQ$	o	$(3,2,4,1)$	$CAP > SQ > ACQ > WAR$
d	$(1,4,2,3)$	$ACQ > WAR > CAP > SQ$	p	$(3,4,1,2)$	$ACQ > SQ > WAR > CAP$
e	$(1,4,3,2)$	$ACQ > CAP > WAR > SQ$	q	$(3,4,2,1)$	$ACQ > SQ > CAP > WAR$
f	$(2,1,3,4)$	$WAR > CAP > SQ > ACQ$	r	$(4,1,2,3)$	$SQ > WAR > CAP > ACQ$
g	$(2,1,4,3)$	$CAP > WAR > SQ > ACQ$	s	$(4,1,3,2)$	$SQ > CAP > WAR > ACQ$
h	$(2,3,1,4)$	$WAR > ACQ > SQ > CAP$	t	$(4,2,1,3)$	$SQ > WAR > ACQ > CAP$
i	$(2,3,4,1)$	$CAP > ACQ > SQ > WAR$	u	$(4,2,3,1)$	$SQ > CAP > ACQ > WAR$
j	$(2,4,1,3)$	$ACQ > WAR > SQ > CAP$	v	$(4,3,1,2)$	$SQ > ACQ > WAR > CAP$
k	$(2,4,3,1)$	$ACQ > CAP > SQ > WAR$	w	$(4,3,2,1)$	$SQ > ACQ > CAP > WAR$

- Traditionally studied strategies
  - Hard defender (m) CAP > SQ > WAR > ACQ
  - Soft defender (o) CAP > SQ > ACQ > WAR
  - Hard challenger (p) ACQ > SQ > WAR > CAP
  - Soft challenger (q) ACQ > SQ > CAP > WAR
  - Rogue challenger (j) ACQ > WAR > SQ > CAP

Table Source: [1]

Motivation

- Are we studying the correct 5 strategies?
- Previous work
  - One player one strategy
  - However, one player can be both – a challenger and a defender
  - Is there a set of optimal strategy pairs?
    - Strategies that ensure that the player survives in the game
  - What are their characteristics?

Modeling of Deterrence Game in our research

Population

P(1,1)	P(1,2)	…	P(1,n)
P(2,1)	P(2,2)	…	P(2,n)
…	…	…	…
P(n,1)	P(n,2)	…	P(n, n)

Strategies : 24 X 24 = 576 strategy pairs

24 challenger strategies

Challenger strategies

24 defender strategies

Defender strategies

Randomly assign

one challenger strategy

one defender strategy

to a player

P(1,1) – $C_{1,1}$, $D_{1,1}$	P(1,2) – $C_{1,2}$, $D_{1,2}$	…	P(1,n) – $C_{1,n}$, $D_{1,n}$
P(2,1) – $C_{2,1}$, $D_{2,1}$	P(2,2) – $C_{2,2}$, $D_{2,2}$	…	P(2,n) – $C_{2,n}$, $D_{2,n}$
…	…	…	…
P(n,1) – $C_{n,1}$, $D_{n,1}$	P(n,2) – $C_{n,2}$, $D_{n,2}$	…	P(n , n) – $C_{n,n}$, $D_{n,n}$

Play the deterrence game – winner replaces loser

Modeling

- Outcome table

Defender strategy

Challenger strategy

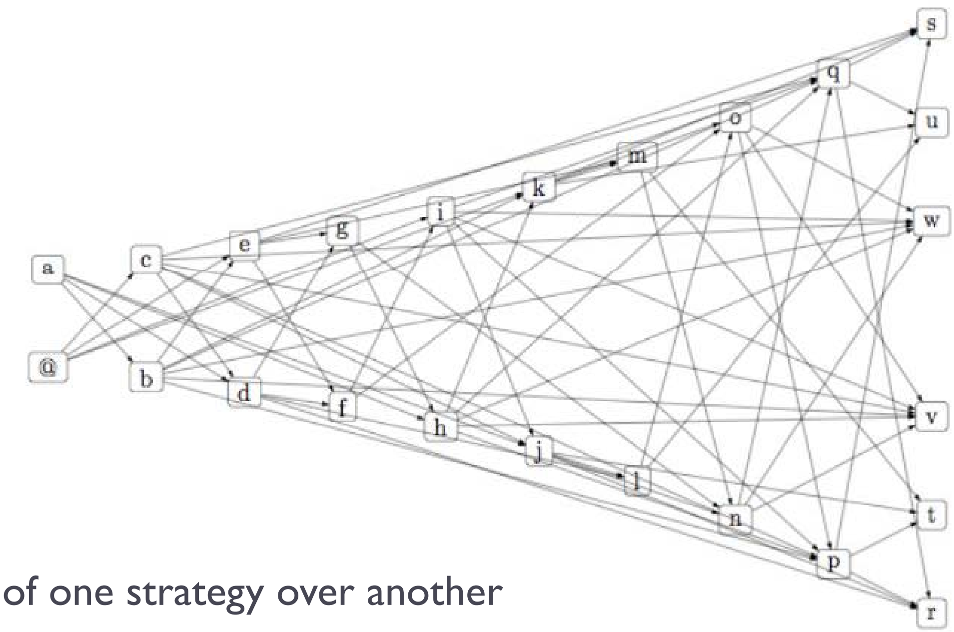
	@	a	b	c	d	e	f	g	h	i	j	k	l	m	n	o	p	q	r	s	t	u	v	w
@	W	W	W	A	A	A	W	W	W	A	A	A	W	W	W	A	A	A	W	W	W	A	A	A
a	C	C	A	C	A	A	C	C	A	C	A	A	C	C	A	C	A	A	C	C	A	C	A	A
b	W	W	W	A	A	A	W	W	W	A	A	A	W	W	W	A	A	A	W	W	W	A	A	A
c	C	C	A	C	A	A	C	C	A	C	A	A	C	C	A	C	A	A	C	C	A	C	A	A
d	W	W	W	A	A	A	W	W	W	A	A	A	W	W	W	A	A	A	W	W	W	A	A	A
e	C	C	A	C	A	A	C	C	A	C	A	A	C	C	A	C	A	A	C	C	A	C	A	A
f	W	W	W	.	.	.	W	W	W	.	.	.	W	W	W	.	.	.	W	W	W	.	.	.
g	C	C	.	C	.	.	C	C	.	C	.	.	C	C	.	C	.	.	C	C	.	C	.	.
h	W	W	W	A	A	A	W	W	W	A	A	A	W	W	W	A	A	A	W	W	W	A	A	A
i	C	C	A	C	A	A	C	C	A	C	A	A	C	C	A	C	A	A	C	C	A	C	A	A
j	W	W	W	A	A	A	W	W	W	A	A	A	W	W	W	A	A	A	W	W	W	A	A	A
k	C	C	A	C	A	A	C	C	A	C	A	A	C	C	A	C	A	A	C	C	A	C	A	A
l	W	W	W	.	.	.	W	W	W	.	.	.	W	W	W	.	.	.	W	W	W	.	.	.
m	C	C	.	C	.	.	C	C	.	C	.	.	C	C	.	C	.	.	C	C	.	C	.	.
n	W	W	W	.	.	.	W	W	W	.	.	.	W	W	W	.	.	.	W	W	W	.	.	.
o	C	C	.	C	.	.	C	C	.	C	.	.	C	C	.	C	.	.	C	C	.	C	.	.
p	.	.	.	A	A	A	.	.	.	A	A	A	.	.	.	A	A	A	.	.	.	A	A	A
q	.	.	A	.	A	A	.	.	A	.	A	A	.	.	A	.	A	A	.	.	A	.	A	A
r	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
s	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
t	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
u	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
v	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
w	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.

  - Outcome of 24 challengers against 24 defenders
  - W – war
  - A – Acquiesce
  - C – Capitulate
  - . – Status Quo

Table Source: [1]

Modeling

- Previous work's acyclic dominance graph for a one player one strategy methodology



  - Based on dominance of one strategy over another
  - Dominance – higher payoff
  - Number of incoming edges determine dominance
  - Winners
    - Status Quo – r, s, t, u, v, w
    - Hard Challenger – p
    - Soft Challenger - q

Graph Source: [1]

Modeling

- Acyclic dominance graph ideal tool to predict winning strategies
  - Winning strategies
    - highest payoff strategies: Status Quo – r, s, t, u, v, w
    - second highest payoff strategies: Hard Challenger – p, Soft Challenger – q

- Present research has 576 strategies
  - 576 vertices in the acyclic dominance graph
  - Is there a dominating set in this dominance graph?
  - This can be found in polynomial time since there are only 576 vertices

Modeling

- Payoff look-up table for 576 strategy pairs

	@	a	b	c	d	e	f	g	h	I	j	k	l	m	n	o	p	q	r	s	t	u	v	w
@	0,0	0,0	0,0	2,0	2,0	2,0	0,0	0,0	0,0	2,0	2,0	2,0	0,0	0,0	0,0	2,0	2,0	2,0	0,0	0,0	0,0	2,0	2,0	2,0
a	0,2	0,2	2,0	0,2	2,0	2,0	0,2	0,2	2,0	0,2	2,0	2,0	0,2	0,2	2,0	0,2	2,0	2,0	0,2	0,2	2,0	0,2	2,0	2,0
b	0,0	0,0	0,0	2,0	2,0	2,0	0,0	0,0	0,0	2,0	2,0	2,0	0,0	0,0	0,0	2,0	2,0	2,0	0,0	0,0	0,0	2,0	2,0	2,0
c	0,2	0,2	2,0	0,2	2,0	2,0	0,2	0,2	2,0	0,2	2,0	2,0	0,2	0,2	2,0	0,2	2,0	2,0	0,2	0,2	2,0	0,2	2,0	2,0
d	0,0	0,0	0,0	2,0	2,0	2,0	0,0	0,0	0,0	2,0	2,0	2,0	0,0	0,0	0,0	2,0	2,0	2,0	0,0	0,0	0,0	2,0	2,0	2,0
e	0,2	0,2	2,0	0,2	2,0	2,0	0,2	0,2	2,0	0,2	2,0	2,0	0,2	0,2	2,0	0,2	2,0	2,0	0,2	0,2	2,0	0,2	2,0	2,0
f	0,0	0,0	0,0	1,1	1,1	1,1	0,0	0,0	0,0	1,1	1,1	1,1	0,0	0,0	0,0	1,1	1,1	1,1	0,0	0,0	0,0	1,1	1,1	1,1
g	0,2	0,2	1,1	0,2	1,1	1,1	0,2	0,2	1,1	0,2	1,1	1,1	0,2	0,2	1,1	0,2	1,1	1,1	0,2	0,2	1,1	0,2	1,1	1,1
h	0,0	0,0	0,0	2,0	2,0	2,0	0,0	0,0	0,0	2,0	2,0	2,0	0,0	0,0	0,0	2,0	2,0	2,0	0,0	0,0	0,0	2,0	2,0	2,0
i	0,2	0,2	2,0	0,2	2,0	2,0	0,2	0,2	2,0	0,2	2,0	2,0	0,2	0,2	2,0	0,2	2,0	2,0	0,2	0,2	2,0	0,2	2,0	2,0
j	0,0	0,0	0,0	2,0	2,0	2,0	0,0	0,0	0,0	2,0	2,0	2,0	0,0	0,0	0,0	2,0	2,0	2,0	0,0	0,0	0,0	2,0	2,0	2,0
k	0,2	0,2	2,0	0,2	2,0	2,0	0,2	0,2	2,0	0,2	2,0	2,0	0,2	0,2	2,0	0,2	2,0	2,0	0,2	0,2	2,0	0,2	2,0	2,0
l	0,0	0,0	0,0	1,1	1,1	1,1	0,0	0,0	0,0	1,1	1,1	1,1	0,0	0,0	0,0	1,1	1,1	1,1	0,0	0,0	0,0	1,1	1,1	1,1
m	0,2	0,2	1,1	0,2	1,1	1,1	0,2	0,2	1,1	0,2	1,1	1,1	0,2	0,2	1,1	0,2	1,1	1,1	0,2	0,2	1,1	0,2	1,1	1,1
n	0,0	0,0	0,0	1,1	1,1	1,1	0,0	0,0	0,0	1,1	1,1	1,1	0,0	0,0	0,0	1,1	1,1	1,1	0,0	0,0	0,0	1,1	1,1	1,1
o	0,2	0,2	1,1	0,2	1,1	1,1	0,2	0,2	1,1	0,2	1,1	1,1	0,2	0,2	1,1	0,2	1,1	1,1	0,2	0,2	1,1	0,2	1,1	1,1
p	1,1	1,1	1,1	2,0	2,0	2,0	1,1	1,1	1,1	2,0	2,0	2,0	1,1	1,1	1,1	2,0	2,0	2,0	1,1	1,1	1,1	2,0	2,0	2,0
q	1,1	1,1	2,0	1,1	2,0	2,0	1,1	1,1	2,0	1,1	2,0	2,0	$\boxed{1,1}$	1,1	2,0	1,1	2,0	2,0	1,1	1,1	2,0	1,1	2,0	2,0
r	1,1	1,1	1,1	1,1	1,1	1,1	1,1	1,1	1,1	1,1	1,1	1,1	1,1	1,1	1,1	1,1	1,1	1,1	1,1	1,1	1,1	1,1	1,1	1,1
s	1,1	1,1	1,1	1,1	1,1	1,1	1,1	1,1	1,1	1,1	1,1	1,1	1,1	1,1	1,1	1,1	1,1	1,1	1,1	1,1	1,1	1,1	1,1	1,1
t	1,1	1,1	1,1	1,1	1,1	1,1	1,1	1,1	1,1	1,1	1,1	1,1	1,1	1,1	1,1	1,1	1,1	1,1	1,1	1,1	1,1	1,1	1,1	1,1
u	1,1	1,1	1,1	1,1	1,1	1,1	1,1	1,1	1,1	1,1	1,1	1,1	1,1	1,1	1,1	1,1	1,1	1,1	1,1	1,1	1,1	1,1	1,1	1,1
v	1,1	1,1	1,1	1,1	1,1	1,1	1,1	1,1	1,1	1,1	1,1	1,1	1,1	1,1	1,1	1,1	1,1	1,1	1,1	1,1	1,1	1,1	1,1	1,1
w	1,1	1,1	1,1	1,1	1,1	1,1	1,1	1,1	1,1	1,1	1,1	1,1	1,1	1,1	1,1	1,1	1,1	1,1	1,1	1,1	1,1	1,1	1,1	1,1

- Example: (q, l) = (1,1) $\Rightarrow$ challenger payoff = 1, defender payoff = 1

Table source: [4]

The Bigger Problem

- In the actual deterrence game, the decision horizon is bounded by an arbitrary number n
  - There could be many attacks and counter-attack
- This implies
  - n decision points
  - n strategies
  - n possibilities
    - n X n payoff look-up table entries
- This changes our dominance graph
  - n vertices
  - m edges

Formal Problem Statement

Theorem: Evolution of dual preference orderings in games of International Conflict is NP-Complete.

- Given:
  - Dominance graph DG(V,E) of evolution of dual preference orderings
  - an integer k, $k \leq |V|$
  - a look-up table L
- Question: Is there a dominating set of ordered pairs in DG of size k or less?

Step 1: Proof of NP

Witness: Dominating set S of ordered pairs

- Use the look-up table
  - First element of ordered pair – Row, Second element – Column
  - Check if the values of the ordered pair in the look-up table are both greater than 0
  - Repeat this for all ordered pairs in the dominating set
  - If all ordered pairs in the dominating set map to non-zero values, then that set is the set of dual preferences.
- Time complexity
  - Look-up: O(1)
  - Check for greater than 0, twice: O(2)
  - Repeat: O(|S|), size of the dominating set
  - Polynomial

Step 2: Proof of NP-Complete

- Dominating Set $\longrightarrow_P$ Evolution of Dual Preference Orderings

- Dominating Set
  - Given: Graph $G = (V,E)$, positive integer $K \leq |V|$.
  - Question: Is there a dominating set of size K or less in G, i.e., a subset $V` \subseteq V$ with $|V`| \leq K$ such that for all $u \in V-V`$ there is a $v \in V`$ for which $\{ u, v \} \in E$?

- Dominating Set is known to be in NP-Complete

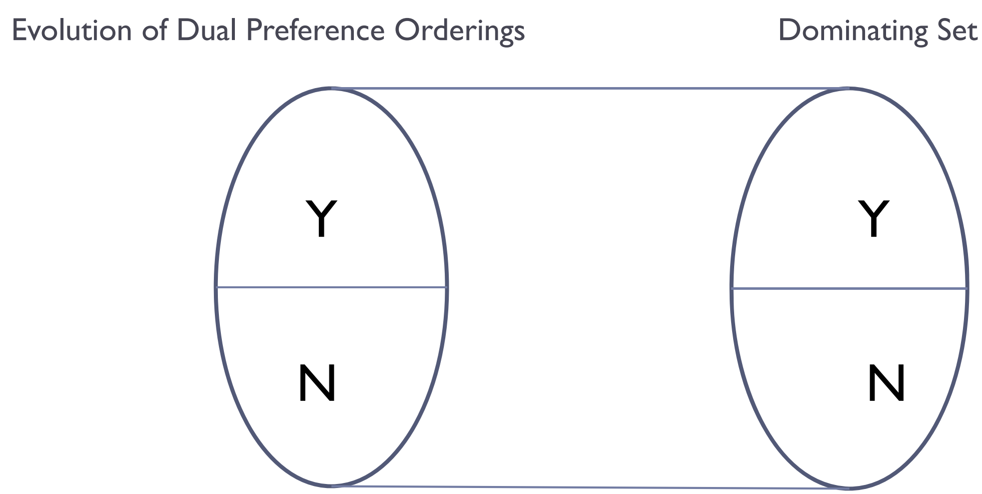
Definition of Dominating Set source: [2]

Proof of Polynomial Transformation

- Step 1: Create an instance of Dominating Set
  - Graph G = (V`,E`)
  - $K \leq |V`|$
- Step 2: Transform this instance to instance of Evolution of dual preference orderings problem
  - Assign G to G``(V``,E``), G`` $\longleftarrow$ G
    - Add a vertex from V` to V ``, if that vertex is in DG(V,E)
      - If a vertex in V is missing in V`, add also that vertex to V``
    - Add an edge from E` to E`` if that edge is in E of DG(V,E)
      - If an edge in E is missing in E`, add also that edge to E``
  - Assign K to k, k$\longleftarrow$K
  - Time complexity: $O(V+E) + O(1) \in O(V+E)$, polynomial

Validation

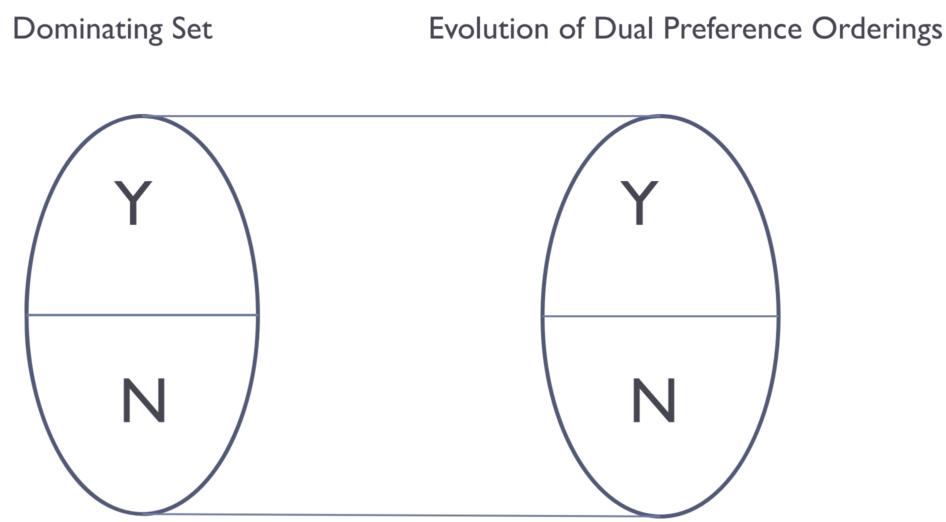
- Step 3:
  - ‘Yes’ instance of evolution of dual preference orderings implies ‘Yes’ instance of Dominating Set



- Step 4: Create an instance of evolution of dual preference orderings problem, i.e., DG(V,E) and k

Validation

- Step 5: Transform it to an instance of Dominating Set G(V`,E`) and K
  - Assign DG to G, G(V`,E`) $\longleftarrow$ DG(V,E) and K $\longleftarrow$ k
  - ‘Yes’ of Dominating Set implies ‘Yes’ of evolution of dual preference orderings



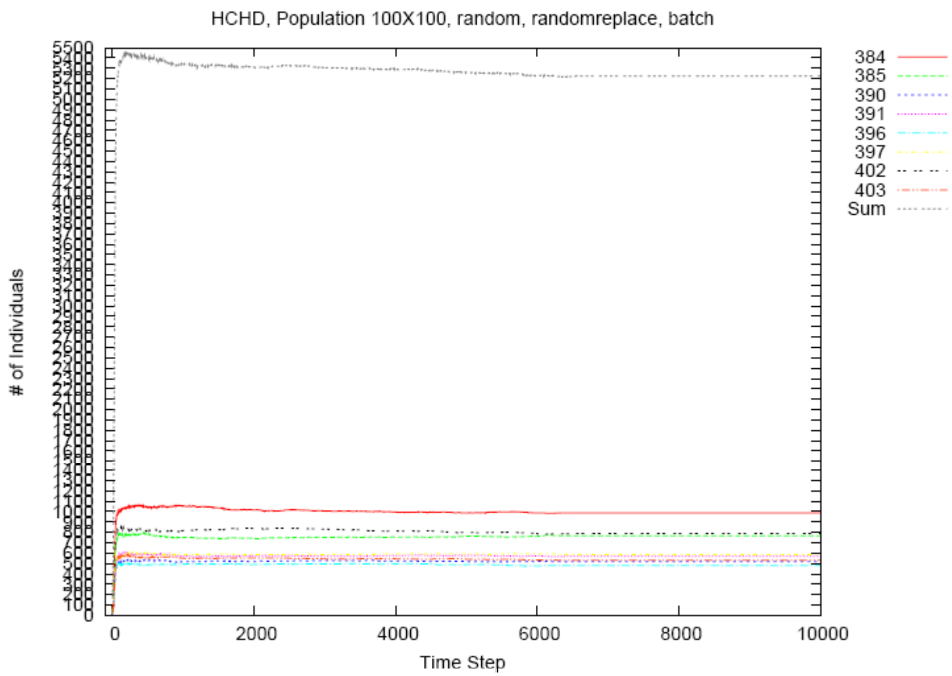
Validation

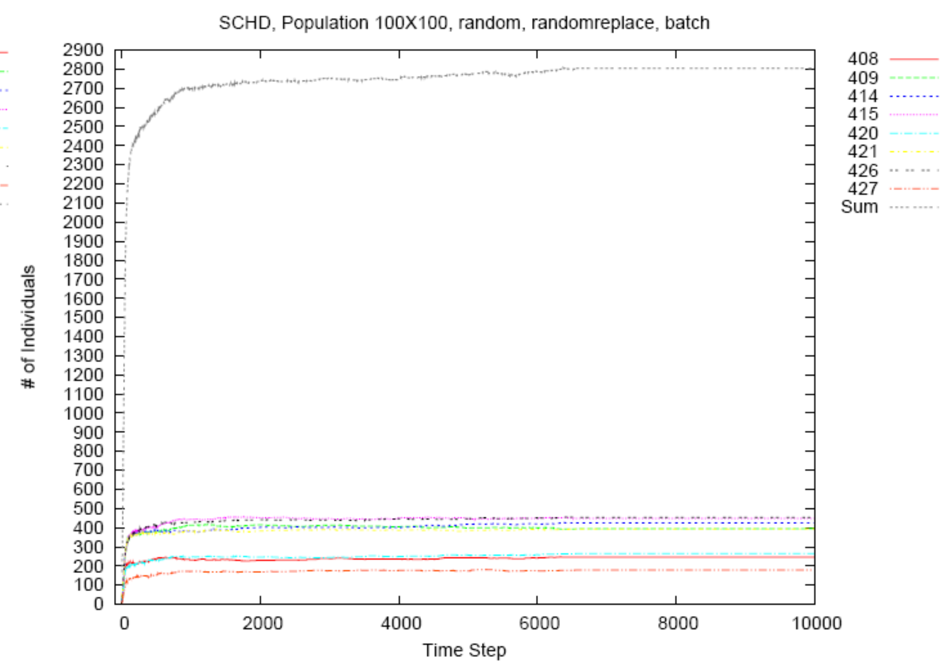
DG(V,E) consists of a dominating set iff G(V,E) consists of a dominating set.

Thus, Evolution of Dual Preference Orderings in Games of International Conflict is NP-Complete.

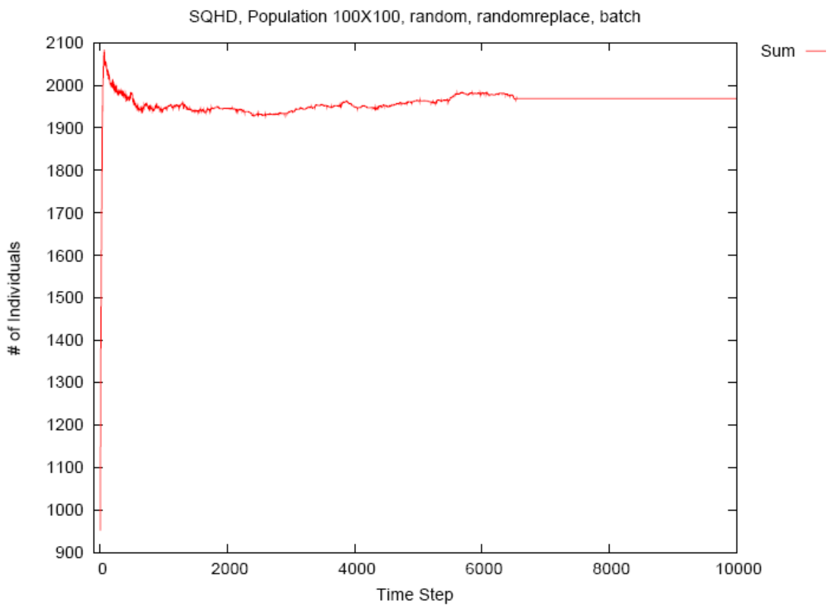
Implications

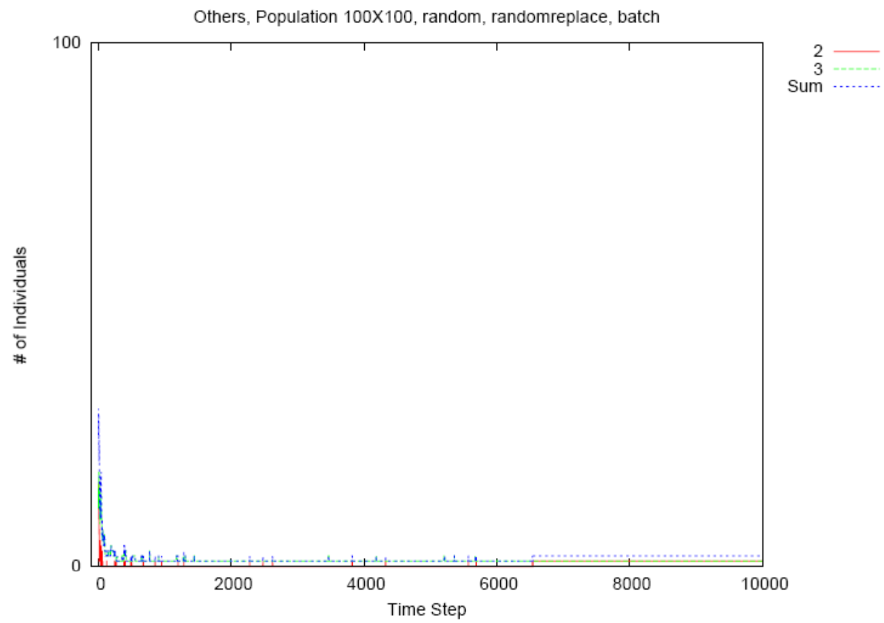
- Survivors in current results
  - Strong challengers paired with strong defenders
  - A few uncommon survivors





Implications





- Will dominance analysis help us understand
  - the approximate 50:25:20 ratio among survivor groups?
  - the presence of uncommon survivors?

- If there is no efficient algorithm to find the solution to this problem, what other methods do we have to use, or how differently should we model this problem?

References

[1] Heckendorn, R. B, Dacey, R., Carlson, L. J, Wu, A. S (2008). '*The Evolution of Preference Orderings in Games of International Conflict'*, presented at the International Studies Association meeting, San Francisco.

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Questions?

Thank you!