Cache Partition Problem for Multi-core System

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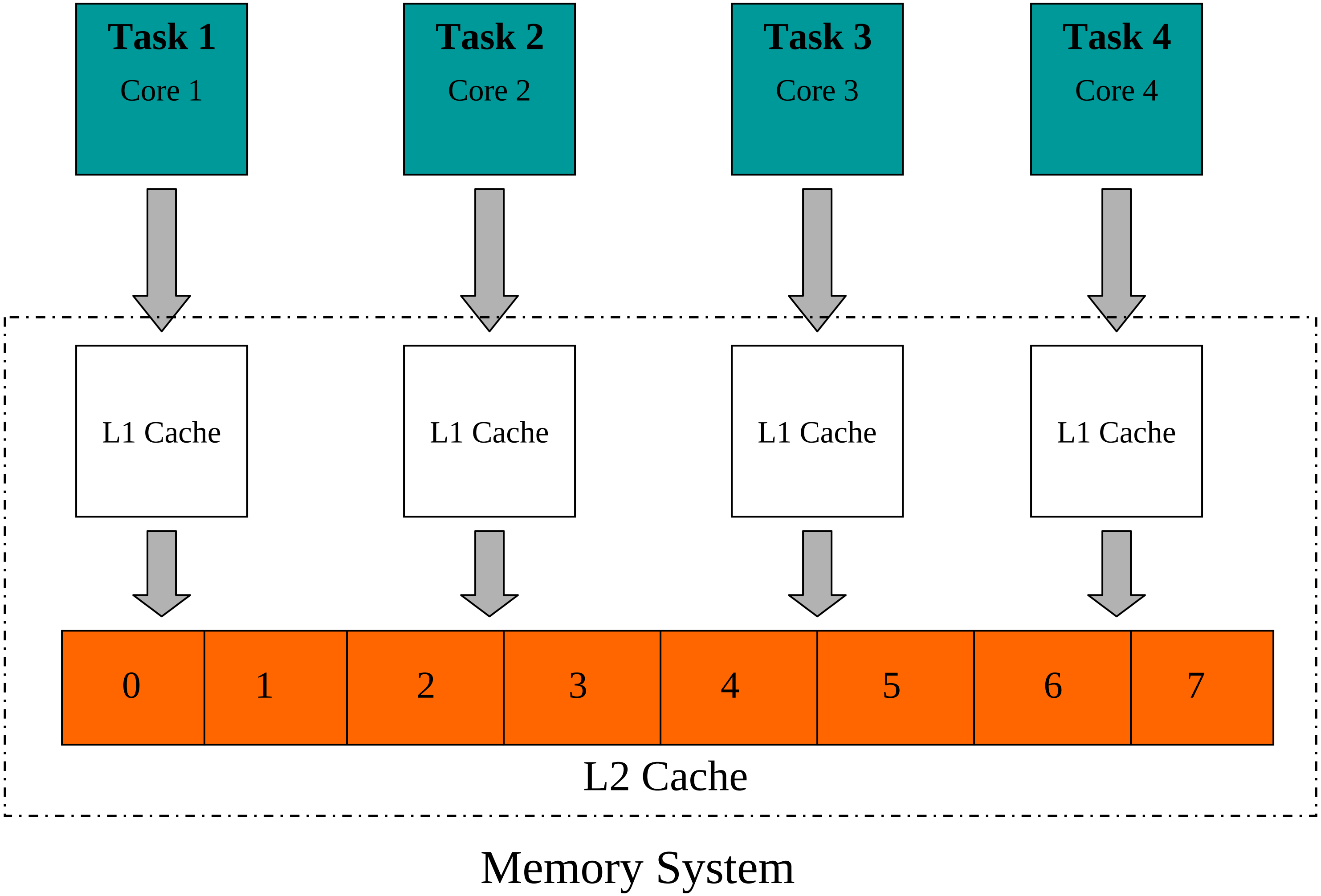
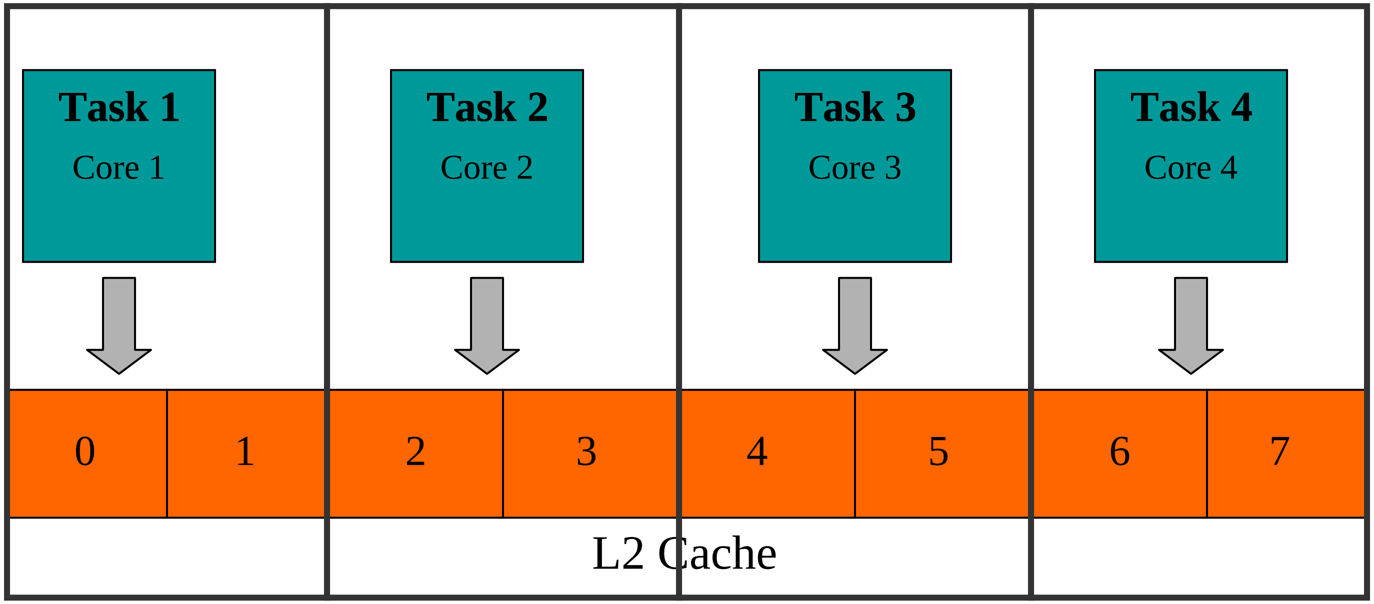
Introduction to the Multi-core System



Introduction to the Multi-core System

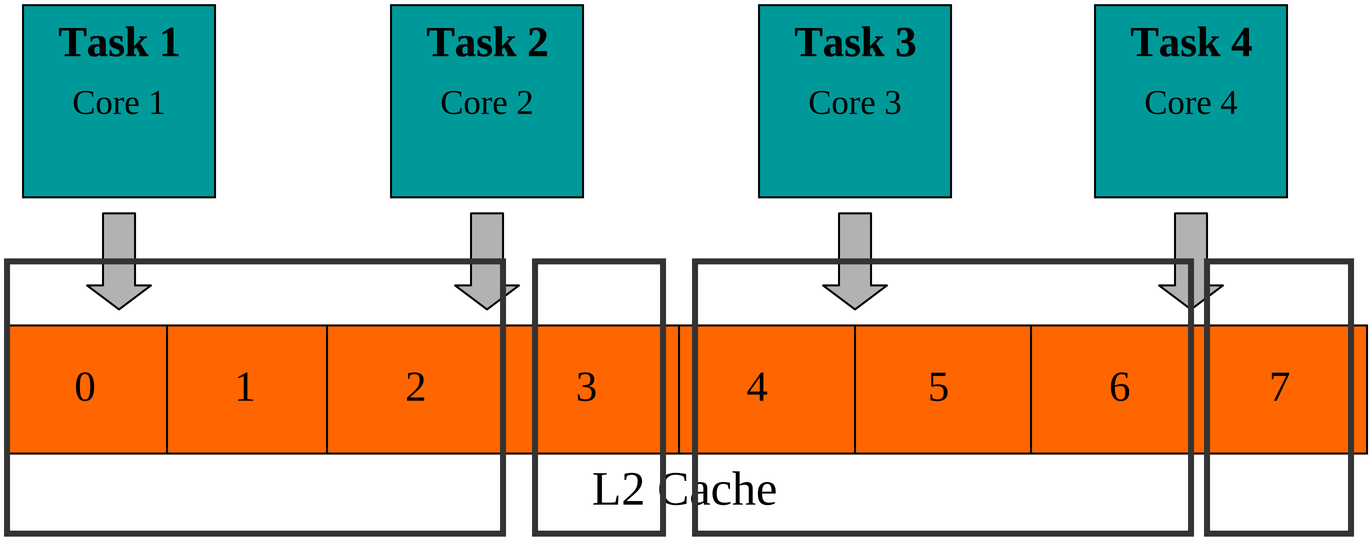


- Application Category:

  -- Cache-sensitive application

  -- Cache-insensitive application

Introduction to the Multi-core System



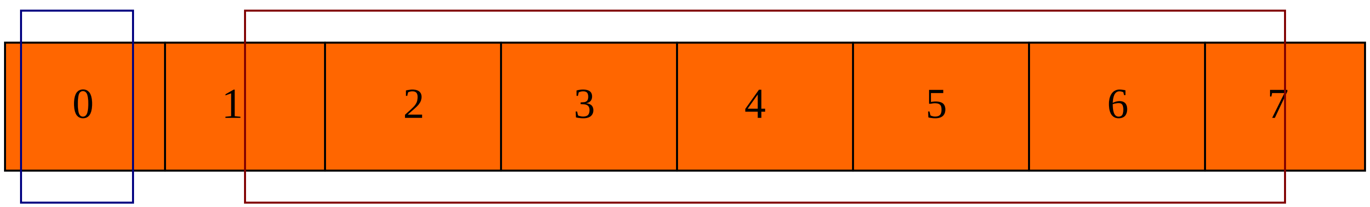
- Property of the task:

  -- Task 1 and Task 3 are cache-sensitive

  -- Task 2 and Task 4 are cache-insensitive

Introduction to the Problem

- WCET: Worst Cache Execution Time
- Given the L2 Cache and tasks on each core, is there any partition which makes ***WCET<m*** ? (Good partition strategy) How?
- Assume there are 2 tasks

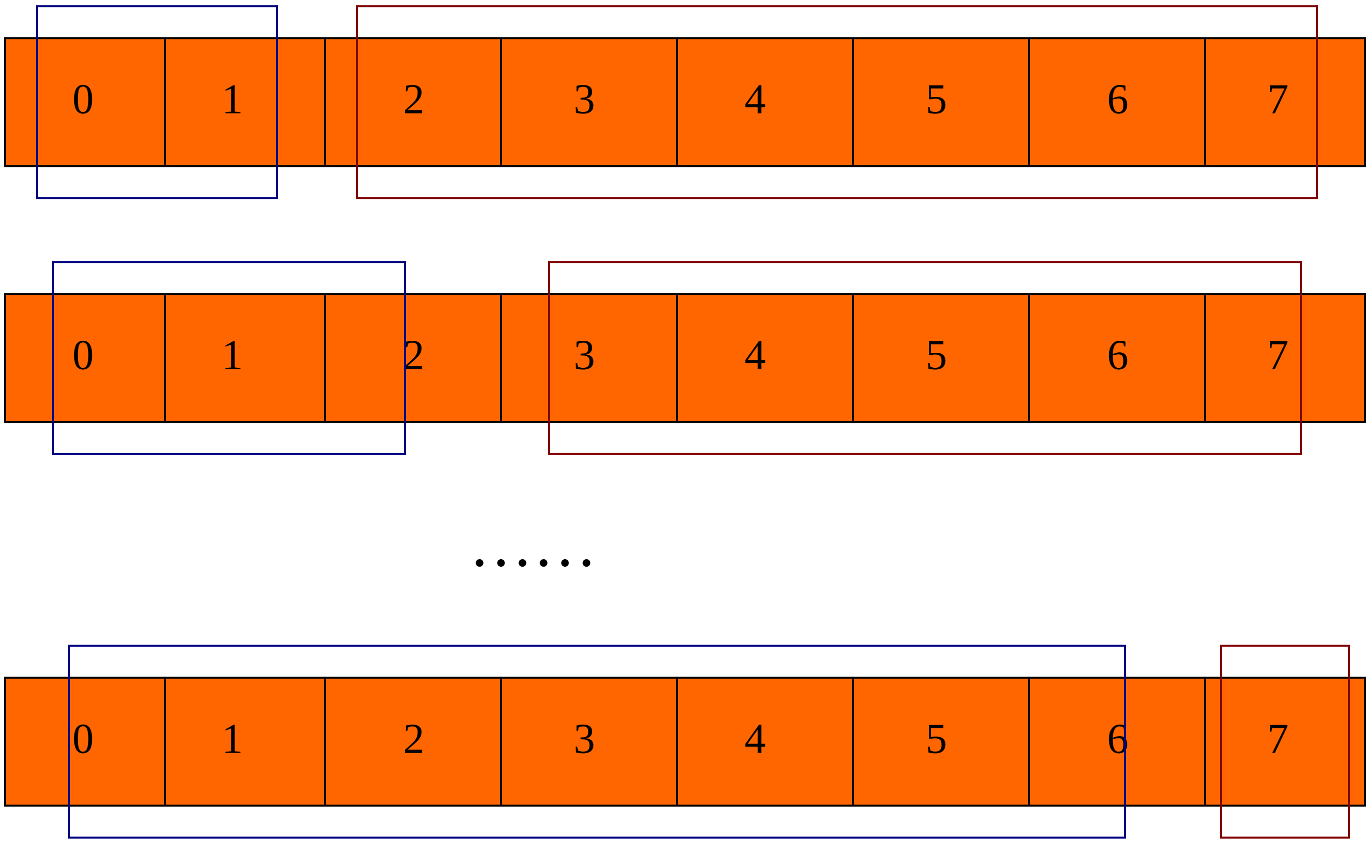


Task1
WCET1

Task12
WCET2

WCET1 + WCET2 < m?

Introduction to the Problem



Task1
WCET1

Task12
WCET2

WCET1 + WCET2 < m? 7 Combinations

Introduction to the Problem

- 4 Tasks on a 32-way L2 Cache?
  -- 6556 Ways

- 8 Tasks on a 32-way L2 Cache?
  -- 15,380,937 Ways

- Greedy Algorithm

Problem Definition

- In a Multi-core System, given:

  m-way L2 Cache,

  n tasks on n cores and the number of each task is executed for C_i times

- Finally there will be n partitions inside the L2 Cache and the corresponding WCET for each partition is $WCET_1, \ldots, WCET_i, \ldots, WCET_n$

- $C_1 * WCET_1 + C_2 * WCET_2 + \ldots + C_n * WCET_n <= T$?

Subset Sum ➡ Multi-core Cache Partition

Problem Reformulation

- *sj* ∈ {*s*1, ..., *sn*}.
- Xi,m denote whether a tasked i has been assigned to partition m

Xi,1 + Xi,2 + ... + Xi,n = 1

- x1,1 * s1 + x1,2 * s1 + ... + x2,1 * s2 + x2,2 * s2 + ... ⩽ m
- WCETi = xi,1 * WCETi,1 + xi,2 * WCETi,2 + ... + xi,n * WCETi,n
- The objective function of the ILP models the WCET of the entire task set for one hyper-period. This overall WCET is defined as:
- WCET = c1 * WCET1 + c2 * WCET2 + ... + cn * WCETn
- The problem finally is:

Does WCET <= T exist? If so, list all the cases.

Problem Reformulation

- Xi,1 + Xi,2 + … + Xi,n = 1
- x1,1 * s1 + x1,2 * s1 + … + x2,1 * s2 + x2,2 * s2 + … ⩽ m
- WCETi = xi,1 * WCETi,1 + xi,2 * WCETi,2 + … + xi,n * WCETi,n
- WCET = c1 * WCET1 + c2 * WCET2 + … + cn * WCETn
- Does WCET <= T exist? If so, list all the cases.

$$WCET = c1 * WCET1 + c2 * WCET2 + \ldots + cm * WCETm$$

$$= c1 * (x1,1 * WCET1,1 + \ldots + x1,n * WCET1,n) + \ldots\ldots$$

$$+ cn * (xn,1 * WCETn,1 + \ldots + xn,n * WCETn,n)$$

$$= \left[c1 * WCET1,1 + \ldots + c1 * WCET1,n + .. + cn * WCETn,1 + \ldots + cn * WCETn,n \right] * \begin{bmatrix} x1,1 \\ \ldots \\ x1,n \\ \ldots \\ xn,1 \\ \ldots \\ xn,n \end{bmatrix} \leq T\ ?$$

Problem Reformulation

(1)

W

$$WCET = \left[c1*WCET_{1,1} + \ldots + c1*WCET_{1,n} + .. + cn*WCET_{n,1} + \ldots + cn*WCET_{n,n}\right] * \begin{bmatrix} x_{1,1} \\ \ldots \\ 1-\sum_{i=1}^{n-1} x_{1,i} \\ \ldots \\ x_{n,1} \\ \ldots \\ 1-\sum_{i=1}^{n-1} x_{n,i} \end{bmatrix} \le T\ ?$$

X

(Given: $X_{i,1} + X_{i,2} + \ldots + X_{i,n} = 1$)

(2)

$$x_{1,1} * s1 + x_{1,2} * s1 + \ldots + x_{2,1} * s2 + x_{2,2} * s2 + \ldots \le m \Rightarrow$$

P

$$[s1, \ldots, s1, s2, \ldots, s2, \ldots, sn, \ldots, sn] * \begin{bmatrix} x_{1,1} \\ \ldots \\ 1-\sum_{i=1}^{n-1} x_{1,i} \\ x_{2,1} \\ \ldots \\ 1-\sum_{i=1}^{n-1} x_{2,i} \\ \ldots \\ x_{n,1} \\ \ldots \\ 1-\sum_{i=1}^{n-1} x_{n,i} \end{bmatrix} \le m$$

Problem Reformulation

- Therefore, based on the previous page, Multi-core Cache Partition problem can be reformulated to:
  $W^T * X \leqslant T$ subject to $P * X \leqslant m$ (X_i can only be 0 or 1)

- The format of this problem looks similar to 0-1 Integer Programming, therefore, we can borrow some idea of the transformation from Subset Sum to 0-1 Integer Programming

Subset Sum to Multi-core Cache Partition

- Multi-core Cache Partition is in the set NP

- Change the Cache Partition problem into slack form:

$$2x1 - 3x2 + 3x3 \geq m\ ?$$
$$Constrains:$$
$$x1 + x2 - x3 \leq 7;$$
$$-x1 - x2 + x3 \leq -7;$$
$$x1 - 2x2 + 2x3 \leq 4;$$
$$x1, x2, x3 \geq 0$$

$\longrightarrow$

$$z = 2x1 - 3x2 + 3x3$$
$$Constraints:$$
$$x4 = 7 - x1 - x2 + x3;$$
$$x5 = -7 + x1 + x2 - x3;$$
$$x6 = 4 - x1 + 2x2 - 2x3;$$
$$x1, x2, x3, x4, x5, x6 \geq 0$$

Subset Sum to Multi-core Cache Partition

- Subset Sum:

  INSTANCE: Finite set A, size s(a) for each a,

  positive integer B.

  QUESTION: Is there a subset A' such that the sum of the sizes of the elements in A' is exactly B?

- Multi-core Cache Partition

  -- Is there any X that satisfies $W^T * X = T$ subject to $P * X = m$ (Xi can only be 0 or 1)

Subset Sum to Multi-core Cache Partition

- Construction:

  Input: F(S,T):

  Output: let S = {S1, S2, …, Sn}, (Constraint: S1+…+Sn = S)

  $$T = X_1*S1 + X_2*S2 + \ldots + X_n*Sn$$

- Suppose that F<S, T> is a yes-instance of subset sum.

  -- A subset of S whose elements can sum to T.

  -- Set corresponding Xi

- Suppose that <s1*X_1 + s2*X_2 + · · · + sn*X_n = T> is a yes instance of Multi-core Cache Partitioning problem

  -- how to set Xi

  -- select a subset of S whose elements sum to T

References

1. Michael R. Garey, David S.Johnson.
   Computers and Intractability: A Guide to the Theory of NP-Completeness, W.H.Freeman and Company, 1979

2. Utility-Based Cache Partitioning: A Low-Overhead, High-Performance, Runtime Mechanism to Partition Shared Caches, Moinuddin K.Qureshi and Yale Patt, the 39th International Symposium on Microarchitecture, 2006

Thank you and Questions?