Frame Building Problem

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Outline

1. Basic Problem
   - Description
   - Examples

2. Proof

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2. Proof

Informal Description

- Need frames to build a greenhouse.
- We have some boards we can use.
- Boards can be cut to make smaller frames.
- Want to minimize extra wood needed.

Formal Definition: *FBP*

Given

A set F of frames $\{f_i\}$ and a set B of boards $\{b_i\} \cup \{E\}$, where E is the length of the extra board.

Question

Does there exist an assignment of every frame to a board such that the sum of the lengths of frames assigned to a board is no greater than the length of the board?

Formal-er Definition: *FBP*

Given

A set F of frames $\{f_i\}$ and a set B of boards $\{b_i\} \cup \{E\}$, where E is the length of the extra board.

Question

Does there exist a total mapping $M : F \longrightarrow B$ such that

$$\forall b_k \in B : \sum_{M(j)=k} f_j \leq b_k$$

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Examples

Example 1

$$B = \{20, 10\} \cup \{5\}$$
$$F = \{10, 5, 4, 3, 3, 3\}$$

Example 2

$$B = \{20, 10\} \cup \{5\}$$
$$F = \{10, 5, 4, 4, 3, 3, 3\}$$

Example 3

$$B = \{20, 10\} \cup \{5\}$$
$$F = \{10, 8, 5, 4, 3, 3, 3\}$$

Failed Approaches

- Multidimensional Knapsack
  - Similar flavor
  - (v_i, w_i) versus f_i
  - Partial versus total mapping
- Zero-One Integer Programming
  - Define the family of indicator variables $a_{i,j}$ that indicate whether f_i is cut from b_j.
  - $\sum_i a_{ij} f_i \leq b_j$ subject to $\sum_{i,j} a_{ij} = \mid F \mid$.

BinPacking[1]

Given

Finite set U of items, a size $s(u) \in Z^+$ for each $u \in U$, a positive integer bin capacity B, and a positive integer K.

Question

Is there a partition of U into disjoint sets $U_1, U_2, \ldots, U_k$ such that the sum of the sizes of the items in each U_i is B or less?

Formal Definition: FBP_K

Given

A set F of frames $\{f_i\}$ and a set B of boards $\{b_i\} \cup \{E\}$, where E is the length of the extra board, and an integer K.

Question

Does there exist an assignment of every frame to a board such that the sum of the lengths of frames assigned to a board is no greater than the length of the board, using at most K boards?

$BinPacking \propto FBP_K$

- Restrict FBP_K to instances where all boards have the same length.
- $b_1 = b_2 = \cdots = b_{|B|}$
- Boards correspond to bins and frames correspond to items.

$FBP_K \propto FBP$

- Reduction from FBP_K to FBP.

Input to FBP_K

- F
- B
- E
- K

Input to FBP

- $F' = F$
- B' ={Largest K elements from B}
- E' =Largest element from B

$FBP_K \propto FBP$

- $Yes(FBP) \rightarrow Yes(FBP_K)$
    - Clearly, this is true, since the created instance of FBP contains exactly K boards.
- $Yes(FBP_K) \rightarrow Yes(FBP)$
    - If the solution to FBP_K uses only elements of B', then this is clearly true.
    - Else, $\exists b_i, b_j \ni b_i$ is used, b_j is not used, $b_i \leq b_j$ and $b_j \in B'$.
    - Then every frame assigned to b_i can be assigned to b_j.
    - By induction, this transforms any solution to a Yes-instance of FBP_K to a Yes-instance of FBP.

Summary

$$BinPacking \propto FBP_K \propto FBP \quad \blacksquare$$

References

Michael R. Garey, David S. Johnson.
Computers and Intractability: A Guide to the Theory of NP-Completeness.
W. H. Freeman and Company, 1979.