Assign#1

Show that prfs are closed under Fibonacci induction. Fibonacci induction means that at each induction step, y+1, after calculating the two base values is computed using the y-th and (y-1)st values. The formal hypothesis is: Assume g1, g2 and h are already known to be prf, then so is f, where f(x,0) = g1(x); f(x,1) = g2(x)f(x,y+1) = h(f(x,y),f(x,y-1))

Hint: The pairing function is useful here.

Let K be the following primitive recursive function, defined by induction on the primitive recursive functions, g, h, and the pairing function.

K(x,0) = < g2(x), g1(x) > K(x, y+1) = J(x, y, K(y,x)) J(x,y,z) = < h(<z>₁, <z>₂), <z>₁ > // this is < f(y+2,x), f(y+1,x)>, even though f is not yet shown to be prf!!

This shows K is prf. f is then defined from K as follows: $f(y,x) = \langle K(y,x) \rangle_2$ // extract second value from pair encoded in K(y,x) This shows it is also a prf, as was desired.

Assign # 2

1. For the sets in a) and b), write a set description that involves the use of a minimum sequence of alternating quantifiers in front of a totally computable predicate (typically formed from **STP** and/or **VALUE**). Choosing from among **(REC) recursive**, **(RE) re non-recursive**, **(Co-RE) co-re**, **non-recursive**, **(HU) non-re/non-co-re**, categorize each of the sets based on the quantified predicate you just wrote. No proofs are required. The following is a sample answer:

Sample.) $S = \{ f | f(x) \text{ diverges for all } x \}$

Co-RE since f is in S iff for all <x,t> [~STP(f, x, t)] a.) A = { f | domain(f) is infinite }

HU since $\forall y \exists \langle x,t \rangle [x \rangle y \& STP(f, x, t)]$

b.) B = { f | f(x) > x for at least one value x } RE since $\exists <x,t> [STP(f, x, t) & VALUE(f,x,t)>x]$

c.) C = { < f,x > | f(x) > x whenever f(x) converges } RE since $\exists t [STP(f, x, t) \Rightarrow VALUE(f,x,t) > x]$

2. Consider the set of indices SemiConstant = { f | |range(f)| = 1 }. Use reduction from Halting Problem to show that SemiConstant is undecidable (not recursive).

Let f,x be arbitrary. Define g(y) = f(x), for all y.

 $\langle f, x \rangle$ is in K₀ iff |range(g)| = 1, as range of g is either 1 (f(x) if $f(x) \downarrow$) or 0 (if $f(x)\uparrow$).

Assign # 3

1. Show that $K \le m$ SemiConstant = { f | |range(f)| = 1 }. To do this, define a total recursive function g, such that index f is in K iff g(f) is in SemiConstant. Be sure to address both cases (f in & f not in).

Let f be arbitrary. Define g(f) = h, where h(y) = f(f), for all y. f is in K iff |range(h)| = 1, as range of g is either 1 (f(f) if $f(f) \downarrow$) or 0 (if $f(f) \uparrow$).

2. What, if anything, does Rice's Theorem have to say about the following? In each case explain by either showing that all of Rice's conditions are met or convincingly that at least one is not met.

a.) SemiConstant = { f | |range(f)| = 1 }

The problem is non-trivial as $C_0(x)$ is in SemiConstant and S(x) = x+1 is not.

The problem is an I/O one, as if two arbitrary function f,g are such that $\forall f(x) = g(x)$ then f is in SemiConstant iff |range(f)| = 1 iff |range(g)| = 1, since range(g) = range(f), iff g is in SemiConstant.

b.) BOUNDED = { f | there is a g such that for all x [if f(x) converges then g(x) > f(x)] } This is trivial since g(x) = f(x) + 1 satisfies the condition for any f. Thus, Rice's theorem says nothing.