Sample Assignment # 5 Key

Consider the Boolean CNF expression **E = (a+b+~c+d+e)(~b)(~a+~d)(b+c+~e)**

1. Recast **E** in 3-CNF form (that is, with each term being a disjunction of three items)

   E=(a+b+f)(~c+d+g)(e+~f+~g)(~b+~b+~b)(~a+~a+~d)(b+c+~e)

2. Present the table that represents a conversion of **E**'s satisfiability to an instance of SubsetSum

	a	b	c	d	e	f	g	a+b+f	~c+d+g	e+~f+~g	~b +~b+~b	~a+~a+~d	b+c+~e
a	1							1					
~a	1											2 (or 1)	
b		1						1					1
~b		1									3 (or 1)		
c			1										1
~c			1						1				
d				1					1				
~d				1								1	
e					1					1			
~e					1								1
f						1		1					
~f						1				1			
g							1		1				
~g							1			1			
C1								1				1	
C1'								1				1	
C2									1				
C2'									1				
C3										1			
C3'										1			
C4											1		
C4'											1		
C5												1	
C5'												1	
C6													1
C6'													1
	1	1	1	1	1	1	1	3	3	3	3	3	3

3. Explicitly write down the numbers that comprise this instance of SubsetSum

1 0 0 0 0 0 0 1 0 0 0 0 0	**a**
1 0 0 0 0 0 0 0 0 0 0 2 0	**~a**
0 1 0 0 0 0 0 1 0 0 0 0 1	**b**
0 1 0 0 0 0 0 0 0 0 3 0 0	**~b**
0 0 1 0 0 0 0 0 0 0 0 0 1	**c**
0 0 1 0 0 0 0 0 1 0 0 0 0	**~c**
0 0 0 1 0 0 0 0 1 0 0 0 0	**d**
0 0 0 1 0 0 0 0 0 0 0 1 0	**~d**
0 0 0 0 1 0 0 0 0 1 0 0 0	**e**
0 0 0 0 1 0 0 0 0 0 0 0 1	**~e**
0 0 0 0 0 1 0 1 0 0 0 0 0	**f**
0 0 0 0 0 1 0 0 0 1 0 0 0	**~f**
0 0 0 0 0 0 1 0 1 0 0 0 0	**g**
0 0 0 0 0 0 1 0 0 1 0 0 0	**~g**
0 0 0 0 0 0 0 1 0 0 0 0 0	**C1**
0 0 0 0 0 0 0 1 0 0 0 0 0	**C1'**
0 0 0 0 0 0 0 0 1 0 0 0 0	**C2**
0 0 0 0 0 0 0 0 1 0 0 0 0	**C2'**
0 0 0 0 0 0 0 0 0 1 0 0 0	**C3**
0 0 0 0 0 0 0 0 0 1 0 0 0	**C3'**
0 0 0 0 0 0 0 0 0 0 1 0 0	**C4**
0 0 0 0 0 0 0 0 0 0 1 0 0	**C4'**
0 0 0 0 0 0 0 0 0 0 0 1 0	**C5**
0 0 0 0 0 0 0 0 0 0 0 1 0	**C5'**
0 0 0 0 0 0 0 0 0 0 0 0 1	**C6**
0 0 0 0 0 0 0 0 0 0 0 0 1	**C6'**
1 1 1 1 1 1 1 3 3 3 3 3 3	**SUBSETSUM**

Show a solution to this SubsetSum instance that encodes a solution to **E**'s satisfiability

1 0 0 0 0 0 0 1 0 0 0 0 0	**a**
0 1 0 0 0 0 0 0 0 0 3 0 0	**~b**
0 0 1 0 0 0 0 0 1 0 0 0 0	**~c**
0 0 0 1 0 0 0 0 0 0 0 1 0	**~d**
0 0 0 0 1 0 0 0 0 0 0 0 1	**~e**
0 0 0 0 0 1 0 0 0 1 0 0 0	**~f**
0 0 0 0 0 0 1 0 1 0 0 0 0	**g**
0 0 0 0 0 0 0 1 0 0 0 0 0	**C1**
0 0 0 0 0 0 0 1 0 0 0 0 0	**C1'**
0 0 0 0 0 0 0 0 1 0 0 0 0	**C2**
0 0 0 0 0 0 0 0 0 1 0 0 0	**C3**
0 0 0 0 0 0 0 0 0 1 0 0 0	**C3'**
0 0 0 0 0 0 0 0 0 0 0 1 0	**C5**
0 0 0 0 0 0 0 0 0 0 0 1 0	**C5'**
0 0 0 0 0 0 0 0 0 0 0 0 1	**C6**
0 0 0 0 0 0 0 0 0 0 0 0 1	**C6'**
1 1 1 1 1 1 1 3 3 3 3 3 3	**SUBSETSUM**

4. Recast the SubsetSum instance you have as an instance of Partition

Add two numbers to set from 3. These are:

3 3 3 3 3 3 3 7 7 7 7 7 7	**2*Sum - G**
3 3 3 3 3 3 3 8 8 8 8 8 8	**Sum + G**

Show an explicit solution to this instance of Partition -- that's easy given (3)

P1:

3 3 3 3 3 3 3 7 7 7 7 7 7	**2*Sum - G**
1 0 0 0 0 0 0 1 0 0 0 0 0	**a**
0 1 0 0 0 0 0 0 0 0 3 0 0	**~b**
0 0 1 0 0 0 0 0 1 0 0 0 0	**~c**
0 0 0 1 0 0 0 0 0 0 0 1 0	**~d**
0 0 0 0 1 0 0 0 0 0 0 0 1	**~e**
0 0 0 0 0 1 0 0 0 1 0 0 0	**~f**
0 0 0 0 0 0 1 0 1 0 0 0 0	**g**
0 0 0 0 0 0 0 1 0 0 0 0 0	**C1**
0 0 0 0 0 0 0 1 0 0 0 0 0	**C1'**
0 0 0 0 0 0 0 0 1 0 0 0 0	**C2**
0 0 0 0 0 0 0 0 0 1 0 0 0	**C3**
0 0 0 0 0 0 0 0 0 1 0 0 0	**C3'**
0 0 0 0 0 0 0 0 0 0 0 1 0	**C5**
0 0 0 0 0 0 0 0 0 0 0 1 0	**C5'**
0 0 0 0 0 0 0 0 0 0 0 0 1	**C6**
0 0 0 0 0 0 0 0 0 0 0 0 1	**C6'**

P2:

3 3 3 3 3 3 3 8 8 8 8 8 8	**Sum + G**
1 0 0 0 0 0 0 0 0 0 0 2 0	**~a**
0 1 0 0 0 0 0 1 0 0 0 0 1	**b**
0 0 1 0 0 0 0 0 0 0 0 0 1	**c**
0 0 0 1 0 0 0 0 1 0 0 0 0	**d**
0 0 0 0 1 0 0 0 0 1 0 0 0	**e**
0 0 0 0 0 1 0 1 0 0 0 0 0	**f**
0 0 0 0 0 0 1 0 0 1 0 0 0	**~g**
0 0 0 0 0 0 0 0 1 0 0 0 0	**C2'**
0 0 0 0 0 0 0 0 0 0 1 0 0	**C4**
0 0 0 0 0 0 0 0 0 0 1 0 0	**C4'**

5. Recast the original SAT instance as an instance of 0-1 Integer Linear Programming

Original SAT: E = (a+b+~c+d+e)(~b)(~a+~d)(b+c+~e)

$0 \le a \le 1$; $0 \le b \le 1$; $0 \le c \le 1$; $0 \le d \le 1$; $0 \le e \le 1$

$a+b+(1-c)+d+e \ge 1$; alternatively, $a+b-c+d+e \ge 0$

$1-b \ge 1$; alternatively, $b = 0$

$(1-a)+(1-d) \ge 1$; alternatively, $a+d \le 1$

$b+c+(1-e) \ge 1$; alternatively, $b+c-e \ge 0$