

Consider the SAT instance:

$(x_1 \vee x_3 \vee \neg x_4 \vee x_5) \& (\neg x_1 \vee x_2 \vee x_3 \vee x_4 \vee \neg x_5) \& (x_1 \vee x_4)$

1. Recast this as an instance of 3SAT.

ANS: $(x_1 \vee x_3 \vee x_6) \& (\neg x_4 \vee x_5 \vee \neg x_6) \& (\neg x_1 \vee x_2 \vee x_7) \& (x_3 \vee \neg x_7 \vee x_8) \& (x_4 \vee \neg x_5 \vee \neg x_8) \& (x_1 \vee x_4 \vee x_1)$

2. Construct the SubsetSum instance equivalent to this and state what rows must be chosen to achieve the desired sum.

ANS:

$c_1 = (x_1 \vee x_3 \vee x_6)$

$c_2 = (\neg x_4 \vee x_5 \vee \neg x_6)$

$c_3 = (\neg x_1 \vee x_2 \vee x_7)$

$c_4 = (x_3 \vee \neg x_7 \vee x_8)$

$c_5 = (x_4 \vee \neg x_5 \vee \neg x_8)$

$c_6 = (x_1 \vee x_4 \vee x_1)$

	x1	x2	x3	x4	x5	x6	x7	x8	C1	C2	C3	C4	C5	C6
x1	1	0	0	0	0	0	0	0	1	0	0	0	0	2
$\sim x1$	1	0	0	0	0	0	0	0	0	0	1	0	0	0
x2	0	1	0	0	0	0	0	0	0	0	1	0	0	0
$\sim x2$	0	1	0	0	0	0	0	0	0	0	0	0	0	0
x3	0	0	1	0	0	0	0	0	1	0	0	1	0	0
$\sim x3$	0	0	1	0	0	0	0	0	0	0	0	0	0	0
x4	0	0	0	1	0	0	0	0	0	0	0	0	1	1
$\sim x4$	0	0	0	1	0	0	0	0	0	1	0	0	0	0
x5	0	0	0	0	1	0	0	0	0	1	0	0	0	0
$\sim x5$	0	0	0	0	1	0	0	0	0	0	0	0	1	0
x6	0	0	0	0	0	1	0	0	1	0	0	0	0	0
$\sim x6$	0	0	0	0	0	1	0	0	0	1	0	0	0	0
x7	0	0	0	0	0	0	1	0	0	0	1	0	0	0
$\sim x7$	0	0	0	0	0	0	1	0	0	0	0	1	0	0
x8	0	0	0	0	0	0	0	1	0	0	0	1	0	0
$\sim x8$	0	0	0	0	0	0	0	1	0	0	0	0	1	0
C1	0	0	0	0	0	0	0	0	1	0	0	0	0	0
C1'	0	0	0	0	0	0	0	0	1	0	0	0	0	0
C2	0	0	0	0	0	0	0	0	0	1	0	0	0	0
C2'	0	0	0	0	0	0	0	0	0	1	0	0	0	0
C3	0	0	0	0	0	0	0	0	0	0	1	0	0	0
C3'	0	0	0	0	0	0	0	0	0	0	1	0	0	0
C4	0	0	0	0	0	0	0	0	0	0	0	1	0	0
C4'	0	0	0	0	0	0	0	0	0	0	0	1	0	0
C5	0	0	0	0	0	0	0	0	0	0	0	0	1	0
C5'	0	0	0	0	0	0	0	0	0	0	0	0	1	0
C6	0	0	0	0	0	0	0	0	0	0	0	0	0	1
C6'	0	0	0	0	0	0	0	0	0	0	0	0	0	1
	1	1	1	1	1	1	1	1	3	3	3	3	3	3

3. Recast the SubsetSum instance in Part 2 as a Partition instance (really easy). Show the Partitioning into equal subsets.

Ans:

G = 11111111333333

sum = 222222255555

2 * sum - G = 333333377777

sum + G = 333333388888

sum is the sum of all rows.

Note: If you use 1 in X1/C6 then

sum is 222222255554 and so

2 * sum - G = 333333377775

sum + G = 333333388887

The partitions for the case where we use 2 in x1/C6 are as follows:

Partition 1:

33333333	777777	2*sum -G
10000000	100002	x1
01000000	001000	x2
00100000	100100	x3
00010000	000011	x4
00001000	010000	x5
00000100	100000	x6
00000010	001000	x7
00000001	000100	x8
00000000	010000	C2
00000000	010000	C2'
00000000	001000	C3
00000000	000100	C4
00000000	000010	C5
00000000	000010	C5'

Partition 2:

33333333	888888	sum+G
10000000	001000	~x1
01000000	000000	~x2
00100000	010000	~x3
00010000	010000	~x4
00001000	000010	~x5
00000100	010000	~x6
00000010	000100	~x7
00000001	000010	~x8
00000000	100000	C1
00000000	100000	C1'
00000000	001000	C3'
00000000	000100	C4'
00000000	000001	C6
00000000	000001	C6'

4. For the 0-1 Integer Linear Programming instance:

$$(x_1 \vee x_3 \vee \neg x_4 \vee x_5) \& (\neg x_1 \vee x_2 \vee x_3 \vee x_4 \vee \neg x_5) \& (x_1 \vee x_4)$$

ANS:

Assume $0 \leq x_1, x_2, x_3, x_4, x_5 \leq 1$

$$x_1 + x_3 + (1-x_4) + x_5 \geq 1$$

$$(1-x_1) + x_2 + x_3 + x_4 + (1-x_5) \geq 1$$

$$x_1 + (1-x_4) \geq 1$$

We choose: $x_1 = 1, x_2 = 1, x_3 = 1, x_4 = 1, x_5 = 1$