Key
for Assign#4

HasExp(HE)={ f | ∃x f(x)↓ & f(x)=2^x }

Show a minimal quantification of some known primitive recursive predicate that provides an upper bound for the complexity of **HE**.

∃<x,t> [STP(f,x,t) & (VALUE(f,x,t) = 2^x]

Thus, **HD $\leq_m$ K_0**

HasExp(HE)={ f | ∃x f(x)↓ & f(x)=2^x }

Use Rice's Theorem to prove that **HD** is undecidable. Be Complete.

HD is non-trivial as **C1(x) = 1** ∈ **HE** as **C1(0) = 1 = 2^0**
and **C0(x)** ∉ **HE**

Let **f,g** be two arbitrary indices of procedures such that
∀**x f(x) = g(x)**

f ∈ **HE** ⇔ ∃**x [f(x)↓ & f(x) = 2^x]**
⇔ ∃**x [g(x)↓ & g(x) = 2^x]**
as ∀**x f(x) = g(x)** and so **g** has same I/O properties
⇔ **g** ∈ **HE**

f ∉ **HE** ⇔ ∀**x [f(x)↓ ⇒ f(x) ≠ 2^x]**
⇔ ∀**x [g(x)↓ ⇒ g(x) ≠ 2^x]**
as ∀**x f(x) = g(x)** and so **g** has same I/O properties
⇔ **g** ∉ **HE**

HasExp(HE)={ f | ∃x f(x)↓ & f(x)=2^x }

Show that **HAS_ID (HI) = { f | ∃x f(x)↓& f(x} = x }** is many-one reducible to **HE**.

Let **f** be an arbitrary index

From **f**, define ∀**x** F_f**(x) = | f(x) - x | +** 2^x

f ∈ **HI** ⇒ **∃x** F_f**(x) =** 2^x ⇒ F_f ∈ **HE**

f ∉ **HI** ⇒ ∀**x [** F_f**(x)**↓ ⇒ F_f**(x)>**2^x **]** ⇒ F_f ∉ **HE**

Thus, **HI** $\leq_m$ **HE**

HasExp(HE)={ f | ∃x f(x)↓ & f(x)=2^x }

Show that **HE** is many-one reducible to
HI = { f | ∃x f(x)↓& f(x} = x }

Let **f** be an arbitrary index

From **f**, define ∀**x** $\mathbf{F_f(x) = | f(x) - 2^x | + x}$
f ∈ **HE** ⇒ **∃x** $\mathbf{F_f(x) = x}$ ⇒ $\mathbf{F_f}$ ∈ **HI**
f ∉ **HE** ⇒ ∀**x [** $\mathbf{F_f(x)}$↓ ⇒ $\mathbf{F_f(x)>x}$ **]** ⇒ $\mathbf{F_f}$ ∉ **HE**

Thus, $\mathbf{HE \leq_m HI}$

IsExp(IE)={ f | ∀x f(x)↓ & f(x)=2^x }

Show a minimal quantification of some known primitive recursive predicate that provides an upper bound for the complexity of **IE**.

∀x ∃t [STP(f,x,t) & (VALUE(f,x,t) = 2^x]

AD looks to be up there with **TOT**

IsExp(IE)={ f | $\forall$x f(x)↓ & f(x)=2^x }

Use Rice's Theorem to prove that **IE** is undecidable. Be Complete.

IE is non-trivial as **Power(x) = 2^x** $\in$ **IE** and **Co(x) = 0** $\notin$ **IE**

Let f,g be two arbitrary indices of procedures such that $\forall$x f(x) = g(x)

f $\in$ **IE** ⇔ $\forall$**x [f(x)**↓ **& f(x) = 2^x]**
⇔ $\forall$**x [g(x)**↓ **& g(x) = 2^x]**
as $\forall$**x f(x) = g(x)** and so **g** has same I/O properties
⇔ **g** $\in$ **IE**

IsExp(IE)={ f | $\forall$x f(x)$\downarrow$ & f(x)=2^x }

Show that **TOT = { f |** $\forall$**x** f**(x)**$\downarrow$ **}** is many-one reducible to **IE**.

Let **f** be an arbitrary index

From **f**, define $\forall$**x** $\mathbf{F_f(x) = f(x) - f(x) + 2^x}$

$\mathbf{f \in TOT \Rightarrow \forall x\ F_f(x) = 2^x \Rightarrow F_f \in IE}$

$\mathbf{f \notin TOT \Rightarrow \exists x\ F_f(x)\uparrow \Rightarrow F_f \notin IE}$

Thus, $\mathbf{TOT \leq_m IE}$

IsExp(IE)={ f | ∀x f(x)↓ & f(x)=2^x }

Show that **IE** is many-one reducible to
TOT = { f | ∀x f**(x)**↓ **}**
Let **f** be an arbitrary index.
From **f**, define ∀**x** $F_f(x)$ = ∃**y [f(x) =** 2^x **]**
f ∈ **IE** ⇒ ∀**x** $F_f(x)$↓ **&** $F_f(x)=1$ ⇒ F_f ∈ **TOT**
f ∉ **IE** ⇒ ∃**x** $F_f(x)$↑ ⇒ F_f ∉ **TOT**.
Thus, **IE** $\leq_m$ **TOT**

That **&** $F_f(x)=1$ is not needed but gives more detail.
Note: It is rare to us**e** ∃ without using **STP** but I am fine with the search failing as a result of **f** diverging on some **x** or failing in a never-ending search.