

Assignment # 6 Key

Consider the Boolean CNF expression $E = (a+b+\sim c+d+e)(\sim b)(\sim a+\sim d)(b+c+\sim e)$

- Recast E in 3-CNF form (that is, with each term being a disjunction of three items)

$$E = (a+b+f)(\sim c+d+g)(e+\sim f+\sim g)(\sim b+\sim b+\sim b)(\sim a+\sim a+\sim d)(b+c+\sim e)$$

- Present the table that represents a conversion of E 's satisfiability to an instance of SubsetSum

	a	b	c	d	e	f	g	a+b+f	$\sim c+d+g$	e+ $\sim f+\sim g$	$\sim b+\sim b+\sim b$	$\sim a+\sim a+\sim d$	b+c+ $\sim e$
a	1							1					
$\sim a$	1											2 (or 1)	
b		1						1					1
$\sim b$		1									3 (or 1)		
c			1										1
$\sim c$			1						1				
d				1					1				
$\sim d$				1								1	
e					1					1			
$\sim e$					1								1
f						1		1					
$\sim f$						1				1			
g							1		1				
$\sim g$							1			1			
C1								1				1	
C1'								1				1	
C2									1				
C2'									1				
C3										1			
C3'										1			
C4											1		
C4'											1		
C5												1	
C5'												1	
C6													1
C6'													1
	1	1	1	1	1	1	1	3	3	3	3	3	3

3. Explicitly write down the numbers that comprise this instance of SubsetSum

1000000100000	a
1000000000020	~a
0100000100001	b
0100000000300	~b
0010000000001	c
0010000010000	~c
0001000010000	d
0001000000010	~d
0000100001000	e
0000100000001	~e
0000010100000	f
0000010001000	~f
0000001010000	g
0000001001000	~g
0000000100000	C1
0000000100000	C1'
0000000010000	C2
0000000010000	C2'
0000000001000	C3
0000000001000	C3'
0000000000100	C4
0000000000100	C4'
0000000000010	C5
0000000000010	C5'
0000000000001	C6
0000000000001	C6'
1111111333333	SUBSETSUM

4. Show a solution to this SubsetSum instance that encodes a solution to E's satisfiability

1000000100000	a
0100000000300	~b
0010000010000	~c
0001000000010	~d
0000100000001	~e
0000010001000	~f
0000001010000	g
0000000100000	C1
0000000100000	C1'
0000000010000	C2
0000000001000	C3
0000000001000	C3'
0000000000010	C5
0000000000010	C5'
0000000000001	C6
0000000000001	C6'
1111111333333	SUBSETSUM

5. Recast the SubsetSum instance you have as an instance of Partition

Add two numbers to set from 3. These are:

3 3 3 3 3 3 7 7 7 7 7 7 2*Sum - G
3 3 3 3 3 3 8 8 8 8 8 8 Sum + G

6. Show an explicit solution to this instance of Partition -- that's easy given (3)

P1:

3 3 3 3 3 3 7 7 7 7 7 7	2*Sum - G
1 0 0 0 0 0 1 0 0 0 0 0	a
0 1 0 0 0 0 0 0 0 3 0 0	~b
0 0 1 0 0 0 0 1 0 0 0 0	~c
0 0 0 1 0 0 0 0 0 0 1 0	~d
0 0 0 0 1 0 0 0 0 0 0 1	~e
0 0 0 0 0 1 0 0 0 1 0 0	~f
0 0 0 0 0 0 1 0 1 0 0 0	g
0 0 0 0 0 0 0 1 0 0 0 0	C1
0 0 0 0 0 0 0 1 0 0 0 0	C1'
0 0 0 0 0 0 0 0 1 0 0 0	C2
0 0 0 0 0 0 0 0 0 1 0 0	C3
0 0 0 0 0 0 0 0 0 1 0 0	C3'
0 0 0 0 0 0 0 0 0 0 1 0	C5
0 0 0 0 0 0 0 0 0 0 1 0	C5'
0 0 0 0 0 0 0 0 0 0 0 1	C6
0 0 0 0 0 0 0 0 0 0 0 1	C6'

P2:

3 3 3 3 3 3 3 8 8 8 8 8 8	Sum + G
1 0 0 0 0 0 0 0 0 0 2 0	~a
0 1 0 0 0 0 0 1 0 0 0 0 1	b
0 0 1 0 0 0 0 0 0 0 0 0 1	c
0 0 0 1 0 0 0 0 1 0 0 0 0	d
0 0 0 0 1 0 0 0 0 1 0 0 0	e
0 0 0 0 0 1 0 1 0 0 0 0 0	f
0 0 0 0 0 0 1 0 0 1 0 0 0	~g
0 0 0 0 0 0 0 0 1 0 0 0 0	C2'
0 0 0 0 0 0 0 0 0 0 1 0 0	C4
0 0 0 0 0 0 0 0 0 0 1 0 0	C4'

7. Recast the 3-CNF form of **E** as an instance of k-Vertex Covering and present a solution to the latter

E=(a+b+f)(~c+d+g)(e+~f+~g)(~b+~b+~b)(~a+~a+~d)(b+c+~e)
Look at notes on the needed gadgets and connections
The k-Vertex cover goal is the number of variables + 2*number of clauses = 7 + 12 = 19.

8. Recast the 3-CNF form of **E** as an instance of k-Coloring and present a solution to the latter

E=(a+b+f)(~c+d+g)(e+~f+~g)(~b+~b+~b)(~a+~a+~d)(b+c+~e)
Look at notes on the needed gadgets and connections. k=3 here.