Assignment #5; Due February 26 at start of class

1. Consider the set of indices **SemiConstant = SC = { f | |range(φ_f)| = 1 }**.

   a) Using **STP**, **VALUE** and a minimum number of alternating quantifiers, describe the set **SemiConstant**.
   $\exists$<x,t>$\forall$<y,s>[STP(f,x,t) & (STP(f,y,s) $\Rightarrow$ VALUE(f,x,t) = VALUE(f,y,s))]}

   b) Show that **K $\le_m$ SemiConstant**, where **K = { f | φ_f(f)$\downarrow$ }**.
   Let f be arbitrary. Define an algorithmic mapping G from indices to indices as G_f (x) = f(f). Now, the range of G_f = {f(f)}. If f is in K, then this range is a singleton value and so G_f is in SC. If f is not in K, then this range is empty and so G_f is not in SC. Thus, K $\le_m$ SC.

   c) Use Rice's Theorem to show that **SemiConstant** is not recursive (not decidable). Note that members of **SemiConstant** do not need to converge for all input, but they must converge on at least one input and when they do converge they always produce the same output value. Hint: There are two properties that must be demonstrated.

   First, SC is non-trivial as Z(x) = 0 is in SC and Z(x) = x is not.

   Second, SC is an I/O Property.
   To see this, let f and g be arbitrary indices of computable functions such that $\forall$x φ_f(x) = φ_g(x).
   f is in SC iff |range(φ_f)| = 1. But g's range is exactly that of f and so, |range(φ_f)| = 1 iff |range(φ_g)| = 1. But then, f is in SC iff g is in SC

   Since SC is not trivial and is an I/O property then it is not recursive by Rice's Theorem.