Assignment #4; Due February 24 at start of class

Choosing from among **(REC) recursive**, **(RE) re non-recursive, (coRE) co-re non-recursive**, **(NRNC) non-re/non-co-re**, categorize each of the sets in a) through d). Justify your answer by showing some minimal quantification of some known recursive predicate.

a.) **$\{ <f, g> \mid \text{domain}(\varphi_f) \subseteq \text{domain}(\varphi_g) \}$**

NRNC

Justification:

$\forall <x,t>\ \exists s\ [STP(f,x,t) \Rightarrow STP(g,x,s)]$

b.) **$\{ f \mid$ no number appears more than once in range$(\varphi_f) \}$**

coRE

Justification:

$\forall <x,y,t>\ [(STP(f,x,t) \& STP(f,y,t) \& (x \neq y)) \Rightarrow (VALUE(f,x,t) \neq VALUE(f,y,t))]$

c.) **$\{ f \mid \varphi_f(f)\downarrow$ in at most f+1 steps $\}$**

REC

Justification:

$STP(f, f, f+1)$

d.) **$\{ f \mid \varphi_f(f)\downarrow$ but takes at least f+1 steps to do so $\}$**

RE

Justification:

$\exists t\ [\ STP(f,f,t)\ \&\ \sim STP(f,f,f)\ ]$

e.) **$\{ <f, x, y> \mid \varphi_f(x)\downarrow$ and $\varphi_f(y)\downarrow$ but $\varphi_f(x)$ takes longer to converge than does $\varphi_f(y) \}$**

RE

Justification:

$\exists t\ [\ STP(f,x,t+1)\ \&\ STP(f,y,t+1)\ \&\ \sim STP(f,x,t)\ ]$