

**Assignment #4; Due February 24 at start of class**

Choosing from among **(REC) recursive**, **(RE) re non-recursive**, **(coRE) co-re non-recursive**, **(NRNC) non-re/non-co-re**, categorize each of the sets in a) through d). Justify your answer by showing some minimal quantification of some known recursive predicate.

a.)  $\{ \langle f, g \rangle \mid \text{domain}(\varphi_f) \subseteq \text{domain}(\varphi_g) \}$

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**Justification:**

Note: This allows equal domains, but even works if  $\text{domain}(\varphi_f)$  is  $\emptyset$  and  $\text{domain}(\varphi_g)$  is  $\mathbb{N}$ .

b.)  $\{ f \mid \text{no number appears more than once in range}(\varphi_f) \}$

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**Justification:**

Note: This can include functions whose ranges are empty and even those whose ranges do include all natural numbers.

c.)  $\{ f \mid \varphi_f(f) \downarrow \text{ in at most } f+1 \text{ steps} \}$

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**Justification:**

Note: This is similar to the set **K** but involves an added twist.

d.)  $\{ f \mid \varphi_f(f) \downarrow \text{ but takes at least } f+1 \text{ steps to do so} \}$

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**Justification:**

Note: This is also similar to **K** but has a twist that differs from that in part (c).

e.)  $\{ \langle f, x, y \rangle \mid \varphi_f(x) \downarrow \text{ and } \varphi_f(y) \downarrow \text{ but } \varphi_f(x) \text{ takes longer to converge than does } \varphi_f(y) \}$

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**Justification:**

Note: Be careful to address the fact that  $\varphi_f$  converges on both **x** and **y**.