

Assignment #3; Due February 17 at start of class

1. Show that prfs are closed under halfway induction. Halfway induction means that each induction step, say at $y+1$, after calculating the base is computed using the value of the function at $\lfloor (y+1)/2 \rfloor$. The formal hypothesis is:

Assume g and h are already known to be prf, then so is f , where

$$f(x,0) = g(x)$$

$$f(x,y+1) = h(f(x, \lfloor (y+1)/2 \rfloor))$$

Define Head by

$$\text{Head}(x) = \langle x \rangle_1$$

Define

$$\text{Tail}(x) = \langle x \rangle_2$$

$$\text{TakeAway}(x,0) = x$$

$$\text{TakeAway}(x,y+1) = \text{Tail}(\text{TakeAway}(x,y))$$

Define F by

$$F(x,0) = \langle g(x), 0 \rangle$$

$$F(x,y+1) = \langle h(\text{Head}(\text{TakeAway}(F(x, y), \lfloor (y+1)/2 \rfloor - 1))), F(x,y) \rangle$$

Now,

$$f(x,y) = \text{Head}(F(x,y))$$

2. Show that prfs are closed under halfway mutual induction. Halfway mutual induction means that each induction step, say at $y+1$, after calculating the base is computed using the value of the other function at $\lfloor (y+1)/2 \rfloor$. The formal hypothesis is:

Assume g_1 , g_2 , h_1 and h_2 are already known to be prf, then so are f_1 and f_2 , where

$$f_1(x,0) = g_1(x); f_2(x,0) = g_2(x)$$

$$f_1(x,y+1) = h_1(f_2(x, \lfloor (y+1)/2 \rfloor)); f_2(x,y+1) = h_2(f_1(x, \lfloor (y+1)/2 \rfloor))$$

$$F(x,0) = \langle \langle g_1(x), 0 \rangle, \langle g_2(x), 0 \rangle \rangle$$

$$F(x,y+1) = \langle \langle h_1(\text{Head}(\text{TakeAway}(\text{Tail}(F(x, y), \lfloor (y+1)/2 - 1 \rfloor))), \text{Head}(F(x,y)) \rangle, \langle h_2(\text{Head}(\text{TakeAway}(\text{Head}(F(x, y), \lfloor (y+1)/2 - 1 \rfloor))), \text{Tail}(F(x,y)) \rangle \rangle$$

Now,

$$f_1(x,y) = \text{Head}(\text{Head}(F(x,y)))$$

$$f_2(x,y) = \text{Head}(\text{Tail}(F(x,y)))$$