

Assignment #3; Due February 17 at start of class

1. Show that prfs are closed under halfway induction. Halfway induction means that each induction step, say at $y+1$, after calculating the base is computed using the value of the function at $\lfloor (y+1)/2 \rfloor$. The formal hypothesis is:

Assume \mathbf{g} and \mathbf{h} are already known to be prf, then so is \mathbf{f} , where

$$\mathbf{f}(x,0) = \mathbf{g}(x)$$

$$\mathbf{f}(x,y+1) = \mathbf{h}(\mathbf{f}(x, \lfloor (y+1)/2 \rfloor))$$

2. Show that prfs are closed under halfway mutual induction. Halfway mutual induction means that each induction step, say at $y+1$, after calculating the base is computed using the value of the other function at $\lfloor (y+1)/2 \rfloor$. The formal hypothesis is:

Assume $\mathbf{g1}$, $\mathbf{g2}$, $\mathbf{h1}$ and $\mathbf{h2}$ are already known to be prf, then so are $\mathbf{f1}$ and $\mathbf{f2}$, where

$$\mathbf{f1}(x,0) = \mathbf{g1}(x); \mathbf{f2}(x,0) = \mathbf{g2}(x)$$

$$\mathbf{f1}(x,y+1) = \mathbf{h1}(\mathbf{f2}(x, \lfloor (y+1)/2 \rfloor)); \mathbf{f2}(x,y+1) = \mathbf{h2}(\mathbf{f1}(x, \lfloor (y+1)/2 \rfloor))$$