Assignment#2; Due February 3 at start of class

Let set **A** be non-empty recursive, **B** be re non-recursive and **C** be non-re. Using the terminology **(REC) recursive**, **(RE) non-recursive recursively enumerable**, **(NR) non-re**, categorize each set below, saying whether or not the set can be of the given category and justifying each answer. You may assume, for any set **S**, the existence of comparably hard sets $\mathbf{S_E} = \{\mathbf{2x|x \in S}\}$ and $\mathbf{S_D} = \{\mathbf{2x+1|x \in S}\}$. The following is a sample of the kind of answer I require:

Sample.) $\mathbf{A \cap C = \{ x \mid x \in A \text{ and } x \in C \}}$

REC: Yes. If $A = \{0\}$ then $A \cap C = \varnothing$ ot $\{0\}$, each of which is in REC.

RE: Yes. Let $A = \aleph_E = \{ 2x \mid x \in \aleph \}$; let $C = TOT_D \cup HALT_E$ then $A \cap C = HALT_E$ which is in RE

NR: Yes. If $A = \aleph$ then $A \cap C = C$, which is in NR.

a.) **$A - B = \{ x \mid x \in A \text{ and } x \notin B \}$ // Set difference**

b.) **$\min(A, B) = \{ \min(x , y) \mid x \in A \text{ and } y \in B \}$ // Minimum**

c.) **$A \oplus C = \{ x \mid x \in A \text{ or } x \in C \text{ but} \notin A \cap C \}$ // Set exclusive union**

Be careful: Some may not be possible. If so, you must justify why this is so.

Note:
TOT = $\{ x \mid \forall\ \varphi_x (y) \downarrow \}$. These are the indices of the set of algorithms.
HALT = $\{ \langle x,y\rangle \mid \varphi_x (y) \downarrow \}$. This is the set of pairs of procedures and input for which the given procedure halts.

The set $\mathbf{S_E}$, for any set **S**, is defined as $\mathbf{\{2x \mid x \in S \}}$
The set $\mathbf{S_D}$, for any set **S**, is defined as $\mathbf{\{ 2x+1 \mid x \in S \}}$.
The complexities of **S**, $\mathbf{S_E}$ and $\mathbf{S_D}$ are the same. That is, all three are either in **REC**, **RE** or **NR**.