

**Assignment#1; Due January 22 at start of class**  
**Review of Formal Languages**

Consider some language  $L$ . For each of the following cases, write in one of (i) through (vi), to indicate what you can say conclusively about  $L$ 's complexity, where

- (i)  $L$  is definitely regular
- (ii)  $L$  is context-free, possibly not regular, but then again it might be regular
- (iii)  $L$  is context-free, and definitely not regular
- (iv)  $L$  might not even be context-free, but then again it might even be regular
- (v)  $L$  is definitely not regular, and it may or may not be context-free
- (vi)  $L$  is definitely not even context-free

Follow each answer with example languages  $A$  (and  $B$ , where appropriate) to back up the complexity claims inherent in your answer; and/or state some known closure property that reflects a bound on the complexity of  $L$ .

**Example.)**  $L = A \cup B$ , where  $A$  and  $B$  are both context free, and definitely not regular

$L$  can be characterized by **Property (ii)**, above.

$L$  is context-free, since the class of context-free languages is closed under union.

$L$  can be regular. For example,

$$A = \{ a^n b^m \mid m \geq n \}, B = \{ a^n b^m \mid m \leq n \},$$

$L = A \cup B = \{ a^n b^m \mid n, m \geq 0 \}$ , which is regular since it can be represented by the regular expression  $a^*b^*$ .

But  $L$  is in general not guaranteed to be regular, e.g., if we just make  $A$  and  $B$  the same context-free, non-regular set, then  $L = A \cup A = A$ , which is not regular.

- a.)  $L = A \cap B$ , where  $A$  and  $B$  are both context-free, non-regular
- b.)  $L = A \cap B$ , where  $A$  is context-free, non-regular and  $B$  is regular
- c.)  $A = B - L$ , where  $A$  is context-free, non-regular and  $B$  is regular
- d.)  $A \subset L$ , where  $A$  is Regular