Numerical Calculus (COT 4500) Final Exam
Date: 4/27/2012

Directions: Please answer each question on your own paper. Show your work. It will be assumed that you used your calculator for basic calculations.

1) (10 pts) Chapter 1: Rounding Errors

(a) (5 pts) Consider calculating the fraction $\frac{a}{b}$, where a is stored accurately as 252 and b is stored as 0.001, but b may be off by a value of 0.0004. What is the minimum value this fraction could be? What is the maximum value?

(b) (5 pts) What are the first 20 bits of the IEEE 754-2008 floating point standard storage of 47.625?

2) (10 pts) Chapter 2, Section 1: Bisection Method

Run 5 iterations of the bisection method searching for a root, x, of the equation $f(x) = x^3 – 3x^2 + x + 1$, where $2 < x < 3$. Please make a chart with columns for both x and f(x). The first value of x you should try is 2.5.

3) (10 pts) Chapter 2, Section 3: Newton's Method

Run 5 iterations of Newton's method searching for a root, x, of the equation $f(x) = x^3 – 3x^2 + x + 1$, where $2 < x < 3$. Please start with $p_0 = 2$ and $p_1 = 3$.

4) (10 pts) Chapter 3, Section 1: Lagrange Polynomial

Find the second Lagrange interpolating polynomial for f(x) = tan x, using $x_0 = .25$, $x_1 = .75$ and $x_2 = 1.25$. Evaluate f(x) rounded to two decimal places for the purposes of this problem.

5) (10 pts) Chapter 4, Section 1: Numerical Differentiation

(a) (6 pts) The strategy used by the Three-Point Midpoint Formula to approximate $f'(x_0)$ is to evaluate the the slope between the points $f(x_0 – h)$ and $f(x_0)$ as well as the slope between the points $f(x_0 + h)$ and $f(x_0)$ and take the average of these two. Prove that simply equals the slope between $f(x_0+h)$ and $f(x_0 – h)$.

(b) (4 pts) Determine a function f with h = .1 such that $f'(x_0)$ differs from what the Three-Point Midpoint Formula approximates as $f'(x_0)$ by more than 25%.

6) (10 pts) Chapter 4, Section 4: Composite Numerical Integration

Use Simpson's Rule with n = 3 to approximate the following definite integral: $\int_{.5}^{1.25} \sin(x^2)\, dx$.

7) (10 pts) Differential Equations

Solve the following differential equation with the initial condition f($\frac{\pi}{4}$) $= 5\sqrt{2}$:

$$ysin(2x) + 2y'sin^2x = 2\sin(x)\cos(2x)$$

Hint: Here are some helpful formulas: $\sin(2x) = 2\sin(x)\cos(x)$, $\cos(2x) = cos^2x - sin^2x$, and $sin^2x + cos^2x = 1$.

8) (10 pts) Chapter 5, Section 2: Euler's Method

Use Euler's method to approximate the solution to the following differential equation with the initial condition y(2) = .5 in the interval 2 ≤ t ≤ 4 and n = 5:

$$y' = 3\sqrt{ty}$$

Please show your approximation for y for each of the five values of t at which we evaluate y during the algorithm.

9) (10 pts) Chapter 6, Section 1: Solving Linear Systems of Equations

Solve the following system of equations in variables a, b, c and d using any method:

$$\begin{gathered} 2a + b + c + d = 4 \\ a + 2b + c + d = 2 \\ a + b + 2c + d = 6 \\ 2a + 2b + 2c + 4d = 11 \end{gathered}$$

10) (10 pts) Fast Fourier Transform

In proving that converting from a point representation to a polynomial representation is the same operation as the original FFT using ω^{-1} as the angle instead, we had to show that the following sum is equal to 0:

$$\sum_{i=0}^{n-1} \omega^{ij}\omega^{ik}$$

where j ≠ n-k and 0<j<n and 0< k< n. Remember that in this context $\omega = \cos\left(\frac{2\pi}{n}\right) + isin(\frac{2\pi}{n})$.

For this question, please prove the assertion in the situation where gcd(j+k, n) = 1. You may also use the fact that $\sum_{i=0}^{n-1} \omega^i = 0$.