COT 4500, Numerical Calculus Assigned: March 30, 2009
S. Lang, Spring 2009 **Assignment #6** (30 pts.) Due: **Wednesday, April 8** in class
**Instruction**: Write your answers clearly and show all relevant work including details. You may use a calculator for the work.
**Note: Show your answers using 8 significant digits with chopping arithmetic.**
**Question 7 added on 4/1/2009. New due date changed on 4/3/2009.**

**Exercise Set for Section 3-4 (pp. 153 – 154):**

1. (6 pts.) Construct the free cubic splines for the following data:

(a)

| $x$ | $f(x)$ |
|---|---|
| –0.15 | -0.19449375 |
| 0.15 | 0.10550625 |

(b)

| $x$ | $f(x)$ |
|---|---|
| –1.0 | 1.1987661 |
| –0.5 | 1.5322807 |
| 0.0 | 2.0000000 |
| 0.5 | 2.4468890 |

2. (4 pts.) Use the cubic splines generated from Question 1 for the given values of $x$ to approximate $f(x)$ and $f'(x)$, and calculate the actual error:
   (a) $f(x) = x^4 - 2x^2 + x$, approximate $f(0)$ and $f'(0)$
   (b) $f(x) = e^x \cos x + 1$, approximate $f(0.25)$ and $f'(0.25)$
3. (6 pts.) Construct the clamped cubic splines using the data of Question 1 and the fact that:
   (a) $f'(-0.15) = 1.5865$ and $f'(0.15) = 0.4135$
   (b) $f'(-1.0) = 0.50832598$ and $f'(0.5) = 0.65644995$

**Exercise Set for Section 4-1 (p.176):**

4. (4 pts.) Use the two-point forward-difference and backward-difference formulas (Eq. 4.1) to determine each missing entry in the following table (for $x = 1.2$ use both forward and backward-difference formulas to approximate $f'(1.2)$):

| $x$ | $f(x)$ | $f'(x)$ |
|---|---|---|
| 1.0 | 1.5403023 | |
| 1.2 | 1.4348293 | (give two approximations) |
| 1.4 | 1.2379540 | |

5. (4 pts.) Compute the actual errors in Question 4 and find the error bounds using the error formulas assuming the function $f(x) = x \cos x + 1$.

**Exercise Set for Section 4-2 (p.185):**

6. (3 pts.) Apply the extrapolation process described in Example 1 of the Notes to determine $N_4(h)$, an approximation to $f'(x_0)$, for the function $f(x) = x \cos x + 1$, $x_0 = 1.0$ and $h = 0.4$.

**Exercise Set for Section 4-3 (p.195):**

7. (3 pts) Approximate the integral $\int_{-0.2}^{0.2} (\cos x)^2 \, dx$ using (a) the Trapezoidal rule, and (b) Simpson's rule.