REGULAR EQUATIONS

Assume that **R**, **Q** and **P** are sets such that **P** does not contain the string of length zero, and **R** is defined by

$$\mathbf{R = Q + RP}$$

We wish to show that

$$\mathbf{R = QP^*}$$

We first show that $\mathbf{QP^*}$ is contained in **R**. By definition, $\mathbf{R = Q + RP}$.
To see if $\mathbf{QP^*}$ is a solution, we insert it as the value of **R** in $\mathbf{Q + RP}$ and see if the equation balances

$$\mathbf{R = Q + QP^*P = Q(e+PP^*) = QP^*}$$

Hence $\mathbf{QP^*}$ is a solution, but not necessarily the only solution.

To prove uniqueness, we show that **R** is contained in $\mathbf{QP^*}$. By definition,

$$\begin{aligned}\mathbf{R} &= \mathbf{Q+RP = Q+(Q+RP)P}\\ &= \mathbf{Q+QP+RP^2 = Q+QP+(Q+RP)P^2}\\ &= \mathbf{Q+QP+QP^2+RP^3}\\ &\mathbf{\ldots}\\ &= \mathbf{Q(e+P+P^2+ \ldots +P^i)+RP^{(i+1)}}, \text{ for all } \mathbf{i>=0}\end{aligned}$$

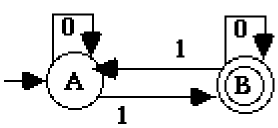
Choose any **W** in **R**, where the length of **W** is equal to **k**. Then, from above,

$$\mathbf{R = Q(e+P+P^2+ \ldots +P^k)+RP^{(k+1)}}$$

but, since **P** does not contain the string of length zero, **W** is not in $\mathbf{RP^{(k+1)}}$. But then **W** is in

$\mathbf{Q(e+P+P^2+ \ldots +P^k)}$ and hence **W** is in $\mathbf{QP^*}$.

We use the above to solve simultaneous regular equations. For example, we can associate regular expressions with finite state automata as follows



$A = \lambda + B1 + A0$

$B = A1 + B0$

Hence,

$$\mathbf{A = B10^* + 0^*}$$

$$\mathbf{B = B10^*1 + B0 + 0^*1}$$

and therefore

$$\mathbf{B = 0^*1(10^*1 + 0)^*}$$

Note: This technique fails if there are lambda transitions.