Closures (Reg. Lang.)

Complement

$$\overline{L} = \{ w \mid w \in \Sigma^* \text{ AND } w \notin L \}$$
$$= \{ w \mid w \in \Sigma^* - L \}$$

Union

$$L_1 \cup L_2 = \{ w \mid w \in L_1 \text{ OR } w \in L_2 \}$$

Intersection

$$L_1 \cap L_2 = \{ w \mid w \in L_1 \text{ AND } w \in L_2 \}$$

Difference

$$L_1 - L_2 = \{ w \mid w \in L_1 \text{ AND } w \notin L_2 \}$$
$$= L_1 \cap \overline{L_2}$$

Exclusive Union

$$L_1 \oplus L_2 = \{ w \mid w \in L_1 \cup L_2 \text{ AND } w \notin L_1 \cap L_2 \}$$
$$= (L_1 - L_2) \cup (L_2 - L_1)$$

DFAs for Closures

Complement

$L = \mathcal{L}(\mathcal{Q})$

$\mathcal{Q} = (Q, \Sigma, \delta, q_0, F)$

$\mathcal{Q}^c = (Q, \Sigma, \delta, q_0, Q-F)$

$\mathcal{L}(\mathcal{Q}^c) = \{w \mid \delta^*(q_0, w) \in (Q-F)\}$

$= \{w \mid \delta^*(q_0, w) \notin F\}$

$= \overline{L}$

DFAs for Closures

Parallel DFAs

$L_1 = \mathcal{L}(Q_1) \qquad L_2 = \mathcal{L}(Q_2)$

$Q_1 = (Q_1, \Sigma, \delta_1, q_0, F_1) \quad Q_2 = (Q_2, \Sigma, \delta_2, s_0, F_2)$

$B = (Q_1 \times Q_2, \Sigma, \delta_3, \langle q_0, s_0 \rangle, F_3)$

WHERE

$\delta_3(\langle q, s \rangle, a) = \langle \delta_1(q, a), \delta_2(s, a) \rangle$

THUS,

$\delta_3^*(\langle q_0, s_0 \rangle, w) = \langle \delta_1^*(q_0, w), \delta_2^*(s_0, w) \rangle$

UNION

$F_3 = F_1 \times Q_2 \cup Q_1 \times F_2$

INTERSECTION

$F_3 = F_1 \times F_2$

DIFFERENCE

$F_3 = F_1 \times (Q_2 - F_2)$

EXCLUSIVE UNION

$F_3 = F_1 \times (Q_2 - F_2) \cup (Q_1 - F_1) \times F_2$

REVERSAL CLOSURE

$L^R = \{w^R \mid w \in L\}$

$\lambda^R = \lambda$

$a^R = a$

$(ax)^R = x^R a$

$L = \mathcal{L}(A)$

$A = (Q, \Sigma, \delta, q_0, F)$

$A^R = (Q, \Sigma, \delta^R, F, \{q_0\})$

WHERE

$\delta^R(q, a) = \{t \mid \delta(t, a) = q\}$

CLEARLY THIS IS <u>NOT</u> A DFA

δ^R MAPS $Q \times \Sigma \to P(Q) = 2^Q$

WE COULD AVOID MULTIPLE START STATES BY INTRODUCING NEW START STATE q_0^R AND STATING

$\delta^R(q_0^R, \lambda) = F$

NDAs

$Q = (Q, \Sigma, \delta, q_0, F)$

$\delta : Q \times \Sigma_e \to P(Q) = 2^Q$

WHERE $\Sigma_e = \Sigma \cup \{\lambda\}$

AND $P(Q)$ IS POWER SET OF Q

FOR CONVENIENCE WE EXTEND

δ TO BE

$\delta : P(Q) \times \Sigma_e \to P(Q)$

WHERE

$\delta(S, a) = \bigcup_{q \in S} \delta(q, a), \quad a \in \Sigma_e$

NOTE:

IF $S = \emptyset$

$\delta(S, a) = \emptyset \quad \forall a \in \Sigma_e$

CLOSURE OF SET OF STATES

$$\lambda\text{-CLOSURE}(S) = \{t \mid t \in \delta^*(S, \lambda)\}$$

$$\delta^*(S, \lambda) = \lambda\text{-CLOSURE}(S)$$

$$\delta^*(S, ax) = \delta^*(\lambda\text{-CLOSURE}(\delta(S, a)), x)$$

$$\mathcal{L}(Q) = \{w \mid \delta^*(\lambda\text{-CLOSURE}(\{q_0\}), w) \cap F \neq \phi\}$$

REGULAR EXPRESSIONS DENOTE REGULAR SETS

REG EXPR		REG SET
$\emptyset$		$\emptyset = \{\ \}$
λ		$\{\lambda\}$
a	$a \in \Sigma$	$\{a\}$
$r_1 + r_2$	WHERE r_1 DENOTES R_1 & r_2 DENOTES R_2	$R_1 + R_2$
$r_1 \cdot r_2$		$R_1 \cdot R_2$
r^*	WHERE r DENOTES R	R^*

COULD SKIP λ AS $\emptyset^* = \{\lambda\}$

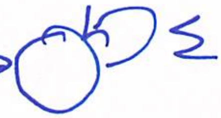
$+$ AS CONVENIENCE

r^+ DENOTES $R^+ = R^* - \{\lambda\}$

$= R^* \cdot R = R \cdot R^*$

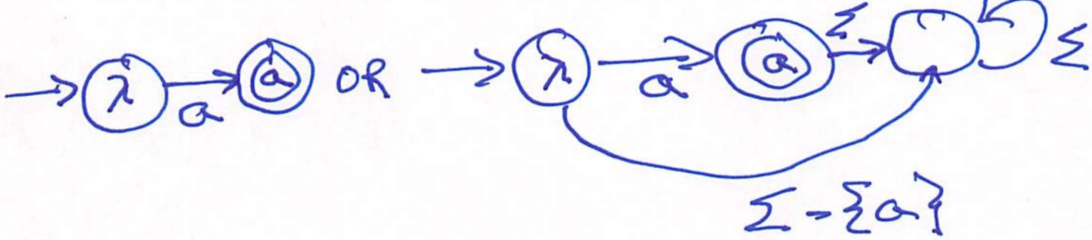
REG. SETS ⊆ REG. LANGS

$\emptyset$

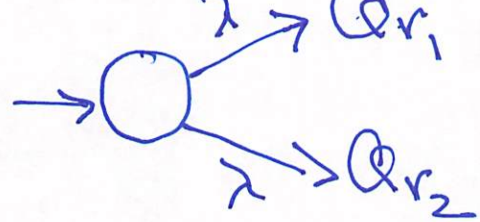


λ

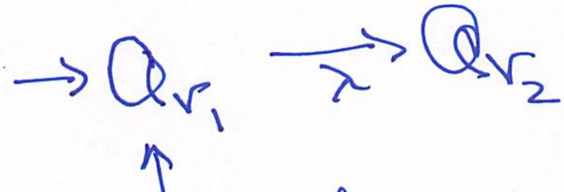
a



$r_1 + r_2$

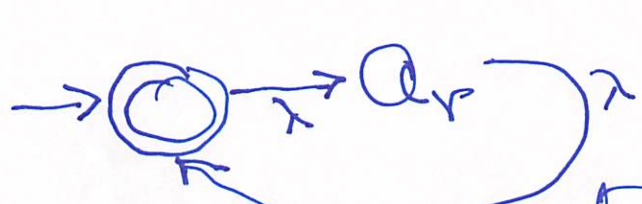


$r_1 \circ r_2$



all final become
non-final but
get transition
$\delta_1(f, \lambda) = q_{0, r_2}$

r^*



only one final = new start
old finals transition on λ
to new start

SAMPLE REG EXPR.

$$(a+b+\dots+z)((a+b+\dots+z)+(0+1+\dots+9))^*$$

$$0^*1(10^*1+0)^*$$

$$(010+11)^+$$

$$(a+bb)^*$$

$$0^*(001+010+11+101)^+$$

REG. LANG. $\subseteq$ REG. SETS

THREE (3) APPROACHES, ALL CAN
START WITH EITHER DFA OR NFA,
EXCEPT REG. EQ. HATE λ- SELF REFERENCES

$$Q = (Q, \Sigma, \delta, q_1, F)$$

WHERE $Q = \{q_1, \ldots, q_n\}$

1. $R_{i,j}^{k}$
2. STATE RIPPING
3. REGULAR EQUATIONS

$$R_{i,j}^{k}$$

$= \{ w \mid \delta^*(q_i, w) = q_j$ AND

NO INTERMEDIATE STATE WITH INDEX $> k$ IS VISITED GOING FROM q_i TO q_j $\}$

HERE

$Q = \{q_1, \ldots, q_n\}$ START IS q_1

AND $R_{i,j}^{k}$ IS SUCH THAT

$1 \leq i, j \leq n$

$0 \leq k \leq n$

$L = \{ w \mid w \in R_{1,f}^{n}$ WHERE $q_f \in F \}$

REALLY WE GET REGULAR EQUATIONS BUT SHOW THAT EXPRESSION DENOTES LANGUAGE

REG. LANG. ⊆ REG. SETS

$R_{i,j}^{k}$

$Q = \{q_1, \ldots, q_n$

$\{q_1\}$ IS START

$1 \le i \le n$

$1 \le j \le n$

$0 \le k \le n$

$R_{i,j}^{k} = \{$expressions | EACH EXPRESSION DENOTES A PATH FROM q_i TO q_j THAT NEVER GOES THROUGH AN INT. STATE WITH INDEX $> k\}$

COMPUTING $R_{i,j}^{k}$

$$R_{i,j}^{0} = \{a \mid \delta(q_i, a) = q_j\}$$
$$\cup \{\lambda \mid i = j\}$$

ACTUALLY REG. EXPR., NOT SETS

$$R_{i,j}^{k+1} = R_{i,j}^{k} + R_{i,k+1}^{k}\left(R_{k+1,k+1}^{k}\right)^{*} R_{k+1,j}^{k}$$

$$R_{L} = \sum_{q_f \in F}^{+} R_{1,f}^{n}$$

EXAMPLE FROM NOTES

STATE RIPPING USAGE

ASSUMES NFA

$Q = (Q, \Sigma, \delta, q_1, F)$

$Q' = (Q \cup \{q_0, f\}, \Sigma, \delta', q_0, \{f\})$

$q_0 \notin Q \quad f \notin Q$

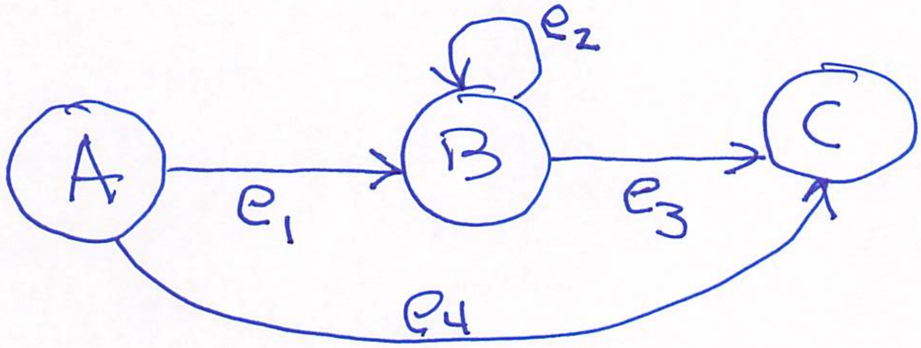
$\delta'(q, a) \supseteq \delta(q, a) \quad q \in Q, a \in \Sigma_e$

$\delta'(q_0, \lambda) = \{q_1\}$

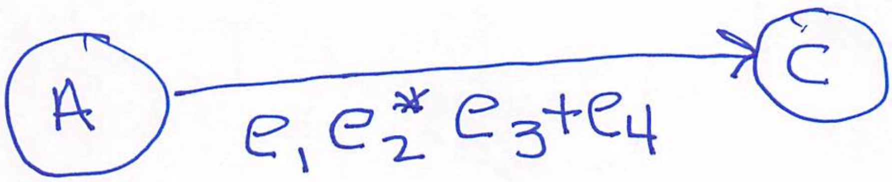
$\delta'(q_f, \lambda) \supseteq \{f\} \quad q_f \in F$

EXAMPLE FROM NOTES

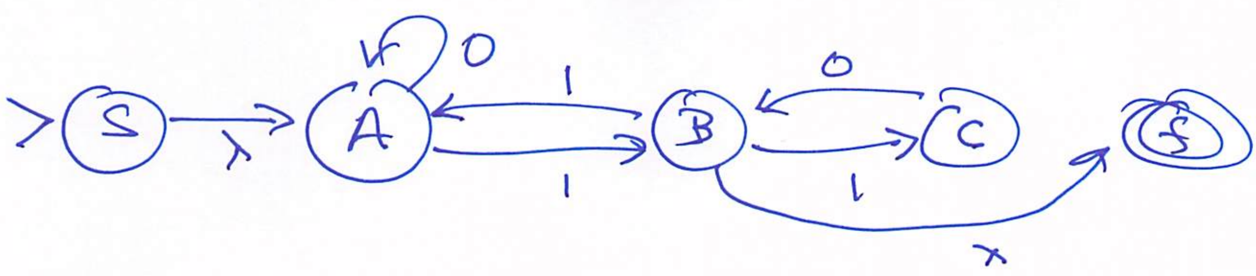
STATE RIPPING



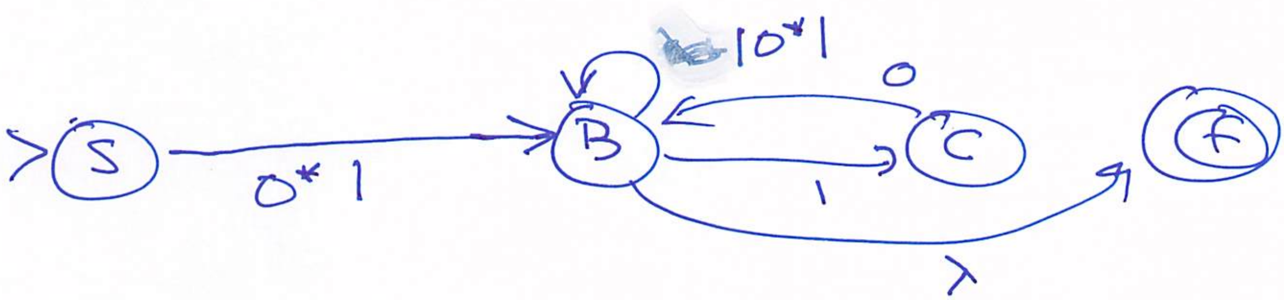
CAN SKIP B BY



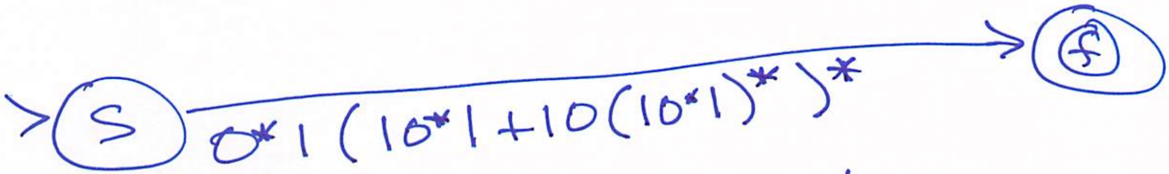
NOTE: A CAN BE SAME AS C



REMOVE A



REMOVE C



$= 0^*1(10^*1+10)^*$

REGULAR EQUATIONS

IF $R = Q + RP$

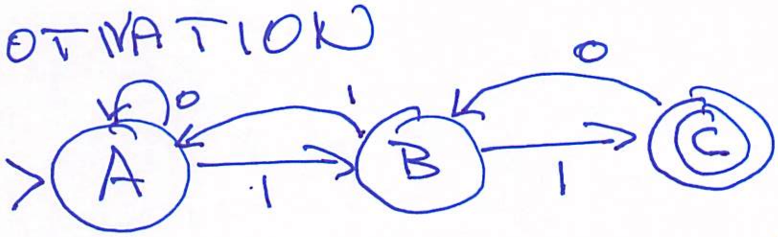
WHERE $\lambda \notin P$ (REALLY λ IS NOT IN SET DENOTED BY P)

THEN
$R = QP^*$ IS THE UNIQUE SOLUTION

THIS IS ARDEN'S THEOREM

PROOF OF UNIQUENESS AND MORE EXAMPLES IN NOTES

MOTIVATION



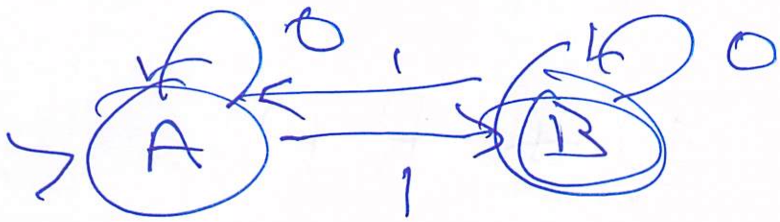
$A = \lambda + A0 + B1$ (RECURSIVE)

$B = A1 + C0$ (INDIRECT RECURSION)

$C = B1$

SOLVING REG. EQ: USING ARDEN'S THEOREM

$$A = (\lambda + B1) + A0$$
$$= (\lambda + B1)0^*$$
$$B = (\lambda + B1)0^*1 + C0$$
$$= (\lambda + B1)0^*1 + B10$$
$$= 0^*1 + B(10^*1 + 10)$$
$$= 0^*1(10^*1 + 10)^*$$



$$A = \lambda + A0 + B1$$

$$B = A1 + B0$$

$$A = (\lambda + B1)0^*$$

$$B = (\lambda + B1)0^*1 + B0$$

$$= 0^*1 + B(10^*1 + 0)$$

$$= 0^*1(10^*1 + 0)^*$$

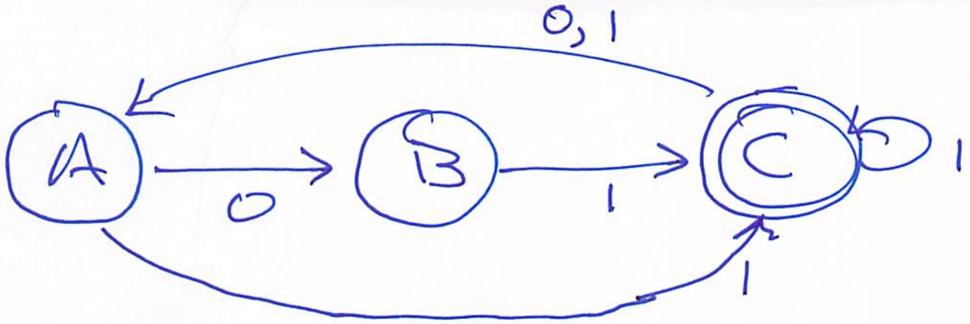
$$R = Q + RP$$

$$R = QP^*$$

$$R = Q + QP^* P$$

$$= Q(\lambda + P^+)$$

$$= QP^*$$



$$A = \lambda + C(0+1)$$

$$B = A0 = 0 + C(0+1)0$$

$$C = A1 + B1 + C\underline{1} = (A1+B1)1^*$$

$$= 1 + C(0+1)1 + 01 + C(0+1)01 + C1$$

$$= (1+01) + C(01+11+001+101+1)$$

$$= (1+01)(01+11+001+101+1)^*$$

SIMPLIFY BY

$$11 = 1 \cdot 1 \subseteq 1^*$$

$$101 = 1 \cdot (01) \subseteq (1+01)^*$$

So $C = L = (1+01)(01+001+1)^*$