

Show  $L = \{ a^i b^j c^k \mid k > \max(i,j), i,j > 0 \}$  is not a CFL

*ME: Assume  $L$  is Context Free*

*PL: Provides a whole number  $N > 0$  that is the value associated with  $L$  based on the Pumping Lemma*

*ME: I chose  $a^N b^N c^{N+1}$  which clearly belongs to  $L$  and has length  $\geq N$ .*

*PL: Breaks  $a^N b^N c^{N+1}$  into five parts  $uvwxy$ , where  $|vwx| \leq N$  and  $|vx| > 0$ . Also, the PL states that  $uv^i wx^i y$  is in  $L$  for all  $i \geq 0$ .*

*Me: Split this into two cases:*

*Case 1:  $vx$  contains at least one  $c$ . Set  $i=0$ , then there are now at most  $N$   $c$ 's since  $|vx| > 0$  and, since  $vwx$  cannot span both  $a$ 's and  $c$ 's, there are still  $N$   $a$ 's. Thus, the  $\max(\#a, \#b) = N$  but then the  $\# c$ 's  $\leq \max(\#a, \#b)$  and hence  $uv^0 wx^0 y = uwy$  is not in  $L$ .*

*Case 2:  $vx$  contains no  $c$ 's. Set  $i=2$ , then there are now at least  $N+1$   $a$ 's or  $N+1$   $b$ 's and so the  $\max(\#a, \#b) \geq N+1$ . However, there are still just  $N+1$   $c$ 's. Thus, the  $\# c$ 's  $\leq \max(\#a, \#b)$  and hence  $uv^2 wx^2 y$  is not in  $L$ .*

*The above cover all cases and so  $L$  is not a CFL.*