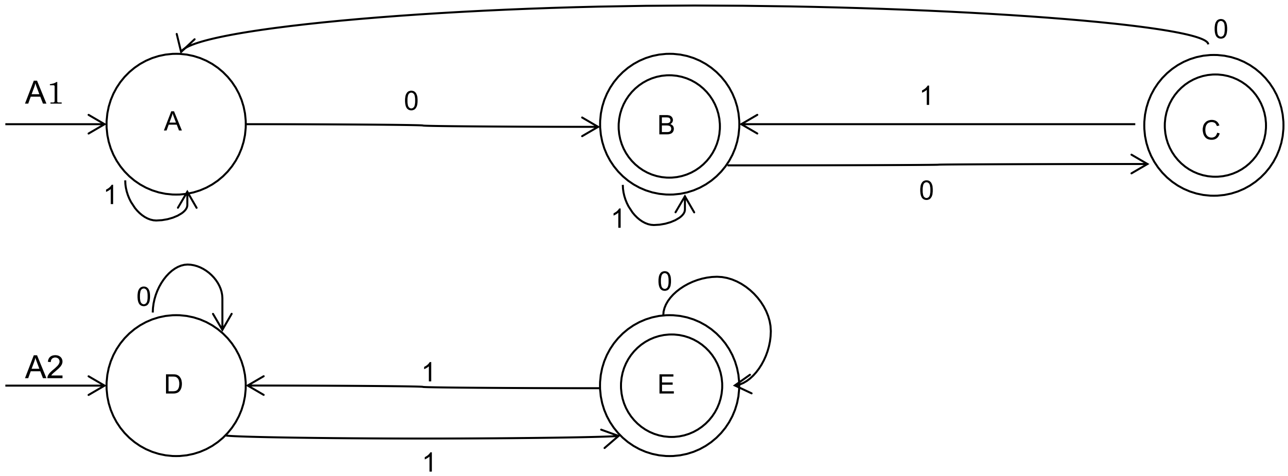
COT 4210 Fall 2019 Midterm#1 Name: KEY

Total Points Available 67 Your Raw Score _______ Grade:____________

6 **1.** Consider the following two DFAs, **A1**, **A2**, using the technique of cross-product of two machines, create a new one that accepts the intersection of ***L*(A1)** and ***L*(A2)**. Be sure to identify the state and final states in your composite DFA. I recommend you use a table here to avoid crossing arcs.



State/Input	*0*	*1*
Start AD	***BD***	***AE***
AE	***BE***	***AD***
BD	***CD***	***BE***
Final BE	***CE***	***BD***
CD	***AD***	***BE***
Final CE	***AE***	***BD***

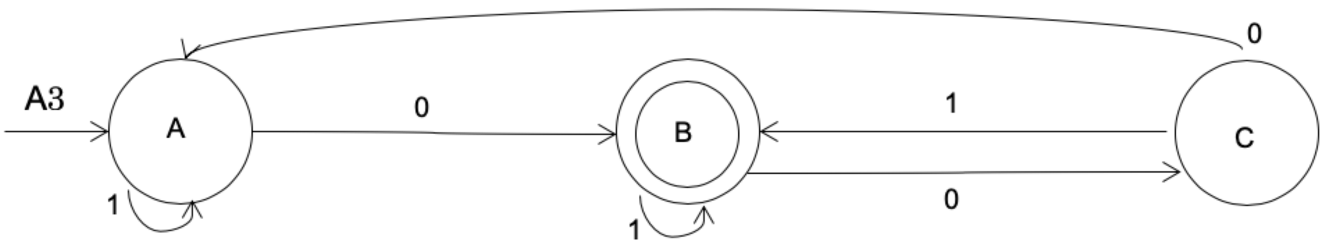
2 **2.** Let **A = ({q1, … q4}, {0,1}, q1, {q2})** be some DFA. Assume you have computed the sets, $\mathbf{R^{k}_{i,j}}$, for $\mathbf{0 \le k \le 3, 1 \le i \le 4, 1, 1 \le j \le 4}$. How do you compute
***L*(A)** = $\mathbf{R^{4}_{1,2}}$, based on the previously computed values of the $\mathbf{R^{k}_{i,j}}$'s?

$R^4_{1,2} = R^3_{1,2} + R^3_{1,4} \cdot (R^3_{4,4})^* \, R^3_{4,2}$

6 **3.** Let ***L1***, ***L2*** be Non-Regular CFLs; ***R*** be Regular; Answer is about ***S*** and there can be more than one cell per row that has an **X**.

Definition of S / Characterization of S	*Can be Regular*	*Can be a non-Regular CFL*	*Can be more complex than a CFL*
S = L1 – L2, where – is set difference	*X*	*X*	*X*
S = L1 • R, where • is concatenation	*X*	*X*	
S = σ(R), where σ is a homomorphism	*X*		
S = σ(L1), where σ is a homomorphism	*X*	*X*	

6 **4.** Let ***L*** be defined as the language accepted by the following DFA **A3**.



Present the regular equations associated with each of **A3**'s states, solving for the regular expression associated with the language recognized by **A3**. You must finish by showing the final expression for the language accepted by this automaton.

A = λ + C0 + A1 = (λ + C0)1* = 1* + C01* = 1* + B001*

B = A0 + C1 + B1 = 1*0 + B001*0 + B01 + B1 = 1*0 (001*0 + 01 + 1)*

C = B0

L = B = 1*0 (001*0 + 01 + 1)*

4 **5.** Looking back at **A3**, above, write a Right Linear Grammar that generates the language accepted by **A3**. Note: You must fill in the list of non-terminals and specify the rules, ***R***.

G = ({ A, B, C }, { 0, 1 }, R, A)

A → 1A | 0B
B → 1B | 0C | λ
C → 0A | 1B

5 **6.** Analyze the language, $L = \{ a^i b^j \mid i < j, j > 0 \}$, proving it is **non**-regular by showing that there are an **infinite** number of equivalence classes formed by the relation $\mathbf{R}_L$ defined by:

$\mathbf{x}\ \mathbf{R}_L\ \mathbf{y}$ if and only if $[\ \forall z \in \{a,b\}^*, xz \in L$ exactly when $yz \in L\ ]$.

You don't have to present all equivalence classes, but you must demonstrate a pattern that gives rise to an infinite number of classes, along with evidence that these classes are distinct.

Consider the right invariant equivalence classes of R_L, $[a^i]_{R_L}$, ***where*** $i \geq 0$.

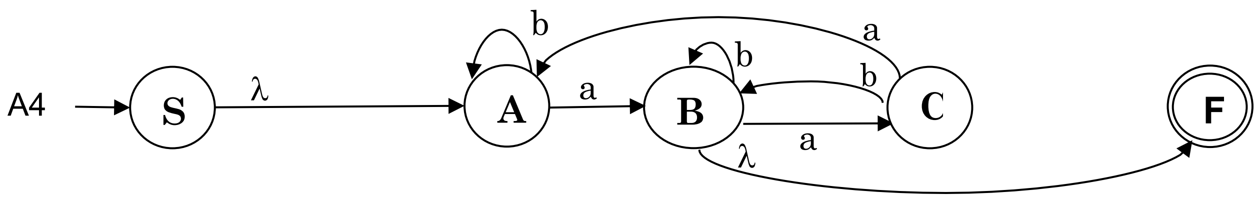
Clearly, $a^i b^{i+1}$ ***is in L***

However, $a^j b^{i+1}$ ***is not in L whenever j>i***

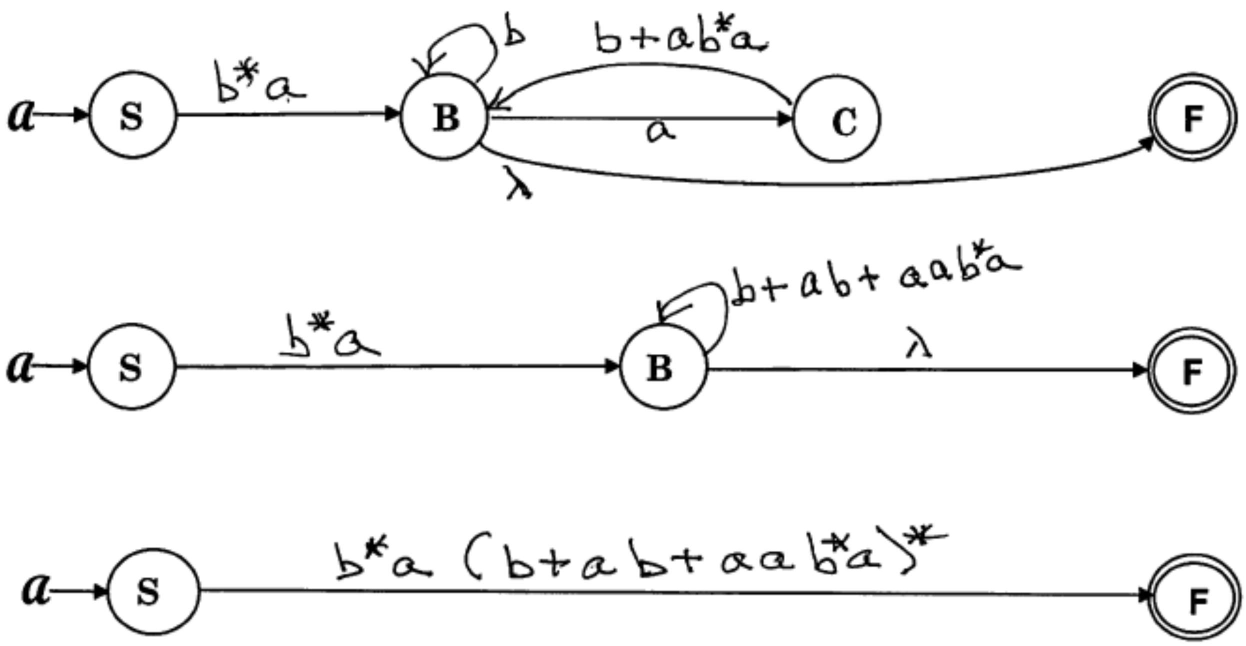
Thus, $[a^i]_{R_L} = [a^j]_{R_L}$ ***iff i=j***

This gives rise to a separate equivalence class for each $i \geq 0$ ***and so*** R_L ***has infinite index and so, by the Myhill-Nerode Theorem, L cannot be Regular.***

6 7. Let **L** be defined as the language accepted by the NFA **A4**:



Using the technique of replacing transition letters by regular expressions and then ripping states from a GNFA to create new expressions, develop the regular expression associated with the automaton **A4** that generates ***L***. I have included the states of a GNFA associated with removing states **A**, **C** and then **B**, in that order. You must use this approach of collapsing one state at a time, showing the resulting transitions with non-empty regular expressions



3 **8.** Define **WeirdSub** (***L***) = { **uv** | **u,v**$\in \Sigma^+$ **and** ($\exists$ **x,y,z** $\in \Sigma^+$) where **xuyvz** $\in$ ***L*** }
Show CFLs are closed under **WeirdSub.** You should find it useful to employ the substitution **f(a) = {a, a'}**, and the homomorphisms **g(a) = a'** and **h(a) = a, h(a') =** λ. Here **a** $\in \Sigma$ and **a'** is a new symbol associated with **a**. You do not need to show your construction works, but it must be based on the meta technique I showed in class.

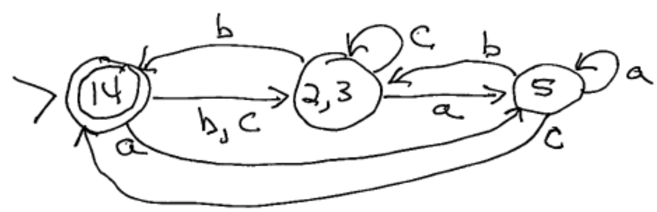
WeirdSub(L) = h (f(L) $\cap$ ***g(***Σ^+***)*** Σ^+ ***g(***Σ^+***)*** Σ^+ ***g(***Σ^+***))***

7 **9.** Given a DFA denoted by the transition table shown below, and assuming that **1** is the start state and **1**, and **4** are final states, fill in the equivalent states matrix I have provided. Use this to create an equivalent, minimal state DFA.

	a	b	c
>**1**	5	3	2
2	5	1	2
3	5	4	3
4	5	2	2
5	5	3	1

2	*X*			
3	*X*	*1,4*		
4	*2,3*	*X*	*X*	
5	*X*	*1,3X* *1,2X*	*3,4X* *1,3X*	*X*
	1	2	3	**4**

Show minimal state equivalent automaton below. Mark the start state and final state(s). Note: if **1, 2** and **3** are indistinguishable, label the merged state as **123**.



6 **10.** Use the Pumping Lemma for CFLs to show that the following language ***L*** is not Context Free.
$L = \{ a^i b^j c^k \mid i < \min(j,k), j,k > 0 \}$.
Be explicit as to why each case you analyze fails to be an instance of ***L*** and, of course, make sure your cases cover all possible circumstances. I have done the first four steps for you and even started the final step by recommending you can split this into two mutually exclusive cases.

ME: Assume L is Context Free

PL: Provides a whole number N>0 that is the value associated with L based on the Pumping Lemma

ME: I chose $a^N b^{N+1} c^{N+1}$ which clearly belongs to L and has length ≥ N.

PL: Breaks $a^N b^{N+1} c^{N+1}$ into five parts uvwxy, where |vwx|≤N and |vx|>0. Also, the PL states that uv^iwx^iy is in L for all i≥0.

Me: Split this into two cases:

Case 1: vx contains at least one a. Set i=2, then there are now at least N+1 a's and at least N+1 b', and, since |vwx|≤N, vwx cannot span both a's and c's, and so there are still N+1 c's. Thus, there are now at least as many a's as c's and hence as the minimum of the b's and c's and so uv^2wx^2y is not in L.

Case 2: vx contains no a's. Set i=0, then there are now either fewer than N+1 b's or fewer than N+1 c's or fewer than N+1 of both. However, there are still N a's. Thus, there are now at least as many a's as the minimum of the b's and c's and hence $uv^0wx^0y = uwy$ is not in L.

The above cover all cases and so L is not a CFL.

11. Demonstrate the steps associated with a Reduced, then Chomsky Normal Form grammar.

3 **a.)** Consider the context-free grammar **G1 = ({ S, A, B }, { 0, 1 }, R1, S)**, where **R1** is:

S → AB1 | BA0
A → 0A0 | B0
B → 1B1 | λ

Remove all λ-rules, except possibly for a start symbol, creating an equivalent grammar **G1**’. Show **all** rules.

Nullable = { B }

S → AB1 | BA0 | A1 | A0
A → 0A0 | B0 | 0
B → 1B1 | 11

3 **b.)** Consider the context-free grammar **G2 = ({ S, A, B }, { 0, 1 }, R2, S)**, where **R2** is

S → AB1 | A
A → 0A0 | B
B → 1B1 | 0

Remove all **unit** rules, creating an equivalent grammar **G2’**. Show **all** rules.

Unit(S) = { S, A, B }; Unit(A) = { A, B }; Unit(B) = { B }

S → AB1 | 0A0 | 1B1 | *0*
A → 0A0 | 1B1 | 0
B → 1B1 | 0

4 **c.)** Consider the reduced context-free grammar **G3 = ({ S, A, B, W, X }, { 0, 1 }, R3, S)**, where **R3** is

S → ABBA | BAAB
A → 0W | 0 **W → A1**
B → 1X | 1 **X → B0**

Convert to an equivalent Chomsky Normal Form grammar **G3’**. Show **all** rules.

S → <AB><BA> | <BA><AB>
A → <0>W | 0 ***W → A<1>***
B → <1>X | 1 ***X → B<0>***
<AB> → AB
<BA> → BA
<0> → 0
<1> → 1

6 **12.** Present the **CKY** recognition matrix for the string **abaab** assuming the Chomsky Normal Form grammar, **G = ({ S, A, B, X, Y, Z }, { a, b }, R, S)**, specified by the rules **R**:.
Note: This matrix is densely populated.

S → AB | BA

A → XY | a

B → XZ | b

Y → AX

Z → BX

X → a | b

	a	b	a	a	b
1	*AX*	*BX*	*AX*	*AX*	*BX*
2	*SY*	*SZ*	*Y*	*SY*	
3	*B*	*A*	*A*		
4	*SZ*	*SY*			
5	*A*				

A little help

Non-Terminal	First Symbol in Rules	Second Symbol in Rules
S	None	None
A	S → AB; Y → AX	S → BA
B	S → BA; Z → BX	S → AB
X	A → XY; B → XZ	Y → AX; Z → BX
Y	None	A → XY
Z	None	B → XZ

Is **abaabb** in **L(G)**? ___*N*___